Anisotropic effects on density information extraction from PS waves (S-Zero Stack)

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Introduction

S-Zero Stack, as a new method to extract robust density information of subsurface from converted waves, has been introduced last year (Zou, 2008). This method separate share velocity effect from density effect analytically in PS reflection equation (Aki & Richard, 1981), thus a special stacking method can be designed to capture density variation at subsurface using PS reflection alone. As common practice of oil industry, this method has been based on a basic assumption of steady-state plan waves propagating in homogeneous layers with small elastic parameter variations at layer interfaces.

In this paper, we will discuss S-Zero Stack under anisotropic consideration. The exact converted wave propagation in anisotropic media is too complex to handle. Plan waves that propagate in transversely isotropic media with weak anisotropy assumption (Thomsen, 1993) are the cases we may considered in this paper. Also, uncertainty in $\Delta\rho/\rho$ estimation using the method has been included in this paper.

S-Zero Stack under anisotropic consideration

VTI model setting give us a simple, but adequate description of layered earth with horizontal formation. The formula of PS reflection at interface of a VTI model was given by Ruger (Ph.D thesis, 1996). Rewrite Ruger's formula using expressions in S-Zero Stack paper (Zou, 2008), we get,

$$R_{PS}^{VTI}(\theta,\gamma) = R_{PS}^{iso}(\theta,\gamma) + R_{PS,VTI}^{anis}(\theta,\gamma) \qquad (1)$$

$$R_{PS}^{iso}(\theta,\gamma) = \frac{Sin\theta}{\gamma\sqrt{\gamma^2 - Sin^2\theta}}\left[2Sin^2\theta - 2Cos\theta\sqrt{\gamma^2 - Sin^2\theta}\right]\frac{\Delta V_S}{V_S} - \frac{Sin\theta}{\gamma\sqrt{\gamma^2 - Sin^2\theta}}\left[\frac{1}{2}\gamma^2 - Sin^2\theta + Cos\theta\sqrt{\gamma^2 - Sin^2\theta}\right]\frac{\Delta\rho}{\rho} \qquad (2)$$

$$R_{PS,VTI}^{anis}(\theta,\gamma,\Delta\delta,\Delta\varepsilon) = \frac{\gamma\sin\theta}{2(\gamma^2-1)}[\sqrt{\gamma^2 - \sin^2\theta} - \cos\theta][\Delta\delta - 2\sin^2\theta(\Delta\delta + \Delta\varepsilon)]$$

here, $\Delta\delta = \delta_2 - \delta_1;\quad \Delta\varepsilon = \varepsilon_2 - \varepsilon_1$ (3)

That is to say, PS reflection coefficient at the subsurface of VTI media $R_{PS}^{VTI}(\theta,\gamma)$, can be separated into two parts, the isotropic contribution $R_{PS}^{iso}(\theta,\gamma)$ and the additional anisotropic contribution $R_{PS,VTI}^{anis}(\theta,\gamma)$.

The anisotropic term $R_{PS,VTI}^{anis}(\theta,\gamma)$ is a function of P incident angle θ, V_P over V_S ratio γ and anisotropic parameter changes $\Delta\varepsilon$, $\Delta\delta$ at the interface, but independent of $\Delta\rho/\rho$ or $\Delta V_S/V_S$. Please note, at each time sample, $\gamma, \Delta\varepsilon$ and $\Delta\delta$ are given, $R_{PS,VTI}^{anis}(\theta,\gamma)$ is a function of P incident angle θ.

To extend further for incident P wave in symmetry-axis plane of HTI media, PS reflection coefficient of Ruger's equation in Appendix D of his thesis can be re-arranged,

$$R_{PS}^{HTI}(\theta,\gamma) = R_{PS}^{iso}(\theta,\gamma) + R_{PS,HTI}^{anis}(\theta,\gamma) \qquad (4)$$

In equation (4), $R_{PS}^{iso}(\theta,\gamma)$ is the same as expressed in equation (2), however $R_{PS,HTI}^{anis}(\theta,\gamma)$ differs from $R_{PS,VTI}^{anis}(\theta,\gamma)$,

$$R_{PS,HTI}^{anis}(\theta) = \frac{\gamma\sin\theta}{2(\gamma^2-1)}[\sqrt{\gamma^2-\sin^2\theta}-\cos\theta][\Delta\delta^V \ -2\sin^2\theta(\Delta\delta^V+\Delta\varepsilon^V)] + \frac{2\Delta\gamma^{anis}}{\gamma}[\sin\theta\cos\theta - \frac{\sin^3\theta}{\sqrt{\gamma^2-\sin^2\theta}}] \qquad (5)$$

$$\Delta\delta^V = \delta_2^V - \delta_1^V;$$
$$\Delta\varepsilon^V = \varepsilon_2^V - \varepsilon_1^V;$$
$$\Delta\gamma^{anis} = \gamma_2^{anis} - \gamma_1^{anis}; \qquad (6)$$

Following Ruger's notation, δ^V and ε^V are Thomsen's weak anisotripic parameter in HTI media defined with respect to vertical in the same way as in VTI media,

$$\varepsilon^V = -\frac{\varepsilon}{1+2\varepsilon}; \quad \delta^V = \frac{\delta - 2\varepsilon(1+\frac{\varepsilon}{f})}{(1+2\varepsilon)(1+\frac{2\varepsilon}{f})}$$

where $f = 1-(V_s/V_P)^2$ (7)

γ^{anis} here is Thomsen's anisotropic parameter for SH waves, added superscript to avoid confusing with γ, which is V_p/V_S used in this paper.

Under above assumption of VTI media or P incident in symmetry-axis plane of HTI media, PS reflection can be written as:

$$R_{PS}^{anis}(\theta,\gamma) = S(\theta,\gamma)\frac{\Delta V_S}{V_S} + D(\theta,\gamma)\frac{\Delta\rho}{\rho} + A(\theta,\gamma),$$

where $A(\theta,\gamma) = R_{PS,VTI}^{anis}(\theta,\gamma)$ or $R_{PS,HTI}^{anis}(\theta,\gamma)$ (8)

The anisotropic term $A(\theta,\gamma)$ is a function of θ,γ and anisotropic parameter variations at the interface, but independent of $\Delta V_S/V_S$ or $\Delta\rho/\rho$. Therefore, the basic equation of S-Zero Stack under VTI or HTI anisotropic assumption can be written as the following,

$$\frac{\Delta\rho}{\rho} = \frac{R_{PS}^{anis}(\theta,\gamma) - A(\theta,\gamma)}{D(\theta,\gamma)}$$

while $S(\theta,\gamma) = 0$ (9)

As examples, Figure 1 shows ratio of anisotropic term $R_{PS,VTI}^{anis}(\theta,\gamma)$ over isotropic term $R_{PS}^{iso}(\theta,\gamma)$ as a function of anisotropic parameters $\Delta\delta$ and $\Delta\varepsilon$ under fixed γ, $\gamma = 2.5$ & 1.7. The horizontal and vertical axis are $\Delta\delta$ and $\Delta\varepsilon$ respectively. The color shows variation of the ratio in percentage. In general, we can see from the diagram, the anisotropic effect enlarges as $\Delta\delta$ and $\Delta\varepsilon$ increase at the interface. However, anisotropic effects increases much more faster along with $\Delta\delta$ than $\Delta\varepsilon$. That is to say anisotropic parameter δ in VTI media dominates anisotropic effect in S-Zero Stack procedure. Also we can see a much larger anisotropic effect for $\gamma = 2.5$ (up panel of Figure 1) than $\gamma = 1.7$ (low panel of Figure 1). Usually γ decreases with depth, comparing two panels in Fig. 1, we can see anisotropic effect in S-Zero Stack, decrease with depth.

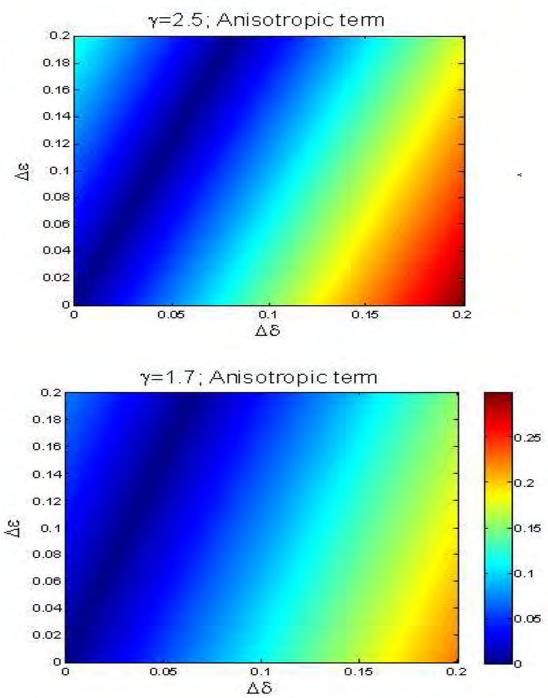

Figure 1, Effects of Anisotropic in S-Zero Stack

Uncertainty in $\Delta\rho/\rho$ estimation

Let's go back to simple isotropic case,

$$\frac{\Delta\rho}{\rho} = \frac{1}{D(\theta,\gamma)}[\mathrm{R}_{PS}(\theta,\gamma) - \mathrm{S}(\theta,\gamma)\frac{\Delta \mathrm{V_S}}{\mathrm{V_S}}] \quad (10)$$

In S-Zero Stack processing, $S(\theta,\gamma) = 0$, however $\frac{\partial S}{\partial\theta} \neq 0$, $\frac{\partial S}{\partial\gamma} \neq 0$, therefore,

$$d(\frac{\Delta\rho}{\rho}) = \frac{1}{D}[\frac{\partial R_{PS}}{\partial\theta}d\theta + \frac{\partial R_{PS}}{\partial\gamma}d\gamma - (\frac{\partial S}{\partial\theta}d\theta + \frac{\partial S}{\partial\gamma}d\gamma)\frac{\Delta V_S}{V_S} - (\frac{\partial D}{\partial\theta}d\theta + \frac{\partial D}{\partial\gamma}d\gamma)\frac{\Delta\rho}{\rho}] \quad (11)$$

Analytical expression of few useful derivatives are shown as follows,

$$\frac{\partial S(\theta,\gamma)}{\partial\theta} = \frac{6\cos\theta\sin^2\theta}{\gamma\sqrt{\gamma^2-\sin^2\theta}} - \frac{2\cos^2\theta}{\gamma} + \frac{2\sin^4\theta\cos\theta}{\gamma(\gamma^2-\sin^2\theta)^{3/2}} + \frac{2\sin^2\theta}{\gamma};$$

$$\frac{\partial S(\theta,\gamma)}{\partial\gamma} = \frac{-2\sin^3\theta}{\gamma^2\sqrt{\gamma^2-\sin^2\theta}} + \frac{2\sin\theta\cos\theta}{\gamma^2} - \frac{2\sin^3\theta}{(\gamma^2-\sin^2\theta)^{3/2}};$$

$$\frac{\partial D(\theta,\gamma)}{\partial\theta} = \frac{\gamma^4\cos\theta - 6\gamma^2\sin^2\theta\cos\theta + 4\sin^4\theta\cos\theta}{-2\gamma(\gamma-\sin\theta)^{3/2}(\gamma+\sin\theta)^{3/2}} + \frac{2(\cos^2\theta-\sin^2\theta)(\gamma-\sin\theta)^{3/2}(\gamma+\sin\theta)^{3/2}}{-2\gamma(\gamma-\sin\theta)^{3/2}(\gamma+\sin\theta)^{3/2}};$$

$$\frac{\partial D(\theta,\gamma)}{\partial\gamma} = \frac{\sin\theta[3\gamma^2\sin^2\theta - 2\sin^4\theta - 2(\gamma-\sin\theta)^{3/2}(\gamma+\sin\theta)^{3/2}\cos\theta]}{-2(\gamma-\sin\theta)^{3/2}(\gamma+\sin\theta)^{3/2}\gamma^2}$$

(12)

Generally, equation (11) shows that uncertainty of estimated $\Delta\rho/\rho$ is a function of the estimated uncertainty of θ and γ in S-Zero Stack. For any common-convert-point of subsurface, uncertainty of estimated $\Delta\rho/\rho$ depends on uncertainty of γ estimation and the way of S-Zero Stack processing. As an example, three point stacking around θ_{S0} is used for a demonstration. The color distribution in Figure 2 represents uncertainty percentage of estimated density contrast. As seen from the diagram, error increases with increasing $\delta\theta$ and $\delta\gamma$ as expected. In this example, we use $\gamma = 2.0$, thus $\theta_{S0} = 63.435^0$, assuming background $\Delta V_S / V_S = 0.2$ and $\Delta\rho/\rho$=0.1. We may vary $\Delta\theta_2$ in S-Zero Stack which represents different stacking procedure. As seen in Figure 2, the smaller $\Delta\theta_2$ is used, the more accurate the density contrast estimation will be, which is opposite to the requirement of using more points in noisy cases. Obviously, there is a tradeoff in $\Delta\theta_2$ selection in S-Zero Stack processing.

Conclusions

We discussed the effect of anisotropic media in S-Zero Stack procedure in this paper and give out the analytical estimation of S-Zero Stack under simple VTI and HTI assumptions. The anisotropic terms can be separated from PS reflection of isotropic medium and treated easily in S-Zero Stack procedure as an additional term. In general, $\Delta\delta$ has larger effects than $\Delta\varepsilon$ and overall anisotropic effect decreases with depth in S-Zero Stack processing.

Acknorledgements

I am grateful to J. Hu (summer intern) for her valuable assistance, to WesternGeco, Schlumberger for permission to publish this paper

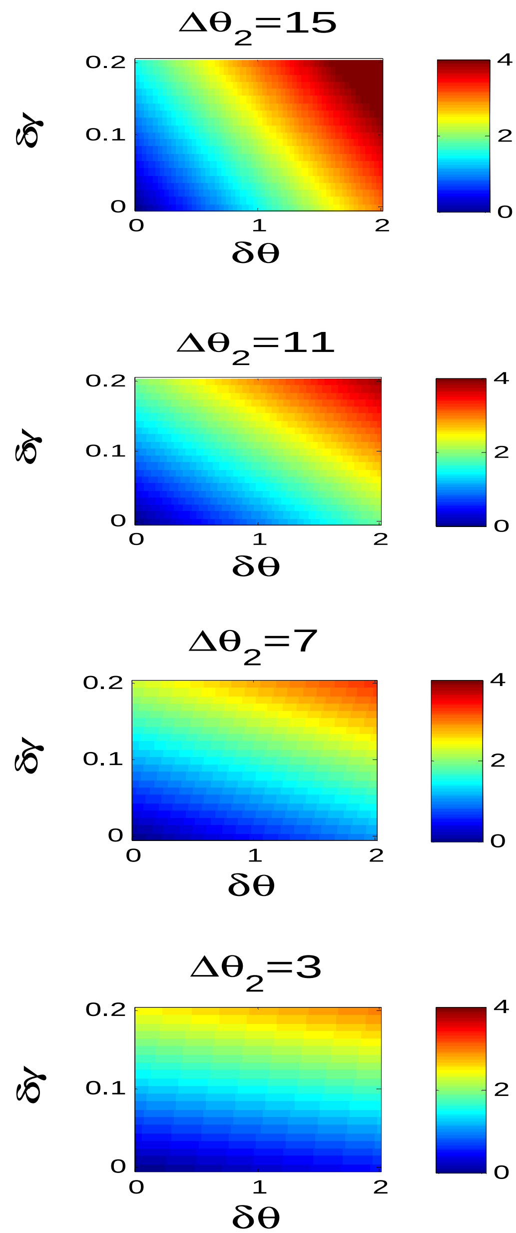

Figure 2, Uncertainty of density contrast estimation using three points stacking around θ_{S0} ($\gamma = 2.0$)

EDITED REFERENCES

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