

Stabilizing acoustic reverse-time migration in TTI media

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Summary

We propose a new stable wave equation for tilted transversely isotropic (TTI) media that can be solved as part of an acoustic anisotropic reverse-time migration (RTM) algorithm using standard explicit finite-differencing. If the shear velocity along the axis of symmetry is set to zero, stable numerical solutions can be computed for vertical transversely isotropic (VTI) media and epsilon not less than delta. In TTI media with rapid variations in the direction of the axis of symmetry, setting the shear velocity along the axis of symmetry to zero can cause numerical solutions to become unstable. A solution to this problem is proposed which involves using a small amount of non-zero shear velocity. The amount of shear velocity added is chosen to remove triplications from the SV wavefront and to minimize the anisotropic term of the SV reflection coefficient. We show stable wave propagation in complex TTI media using this equation.

Introduction

Seismic anisotropy is observed in many exploration areas. Conventional isotropic RTM for seismic imaging is insufficient in these areas. Rather than solving the complicated anisotropic elastic wave equation, many approaches have derived simpler two-way wave equations that can be efficiently solved to perform acoustic anisotropic RTM of pressure data.

Alkhalifah (1998) introduced a pseudo-acoustic approximation for transversely isotropic media, by setting the shear velocity along the axis of symmetry to zero. Although this dispersion relation for a scalar wavefield has kinematics that are close to those of the P arrivals in the real elastic vector wavefield, it allows spurious SV events (Grechka et al., 2004). Many researchers have derived and implemented two-way wave equations for vertical transversely isotropic media starting from Alkhalifah's dispersion relation (Alkhalifah, 2000; Klie and Toro, 2001; Zhou et al., 2006a; Hestholm, 2007; Du et al., 2008). Alternatively, starting from Hooke's law and the equations of motion, vertical shear velocity can be set to zero to derive a pseudo-acoustic VTI wave equation (Duveneck et al., 2008). Tilting the symmetry axis relative to the coordinates does not add any new physics, just more algebraic complexity. Several pseudo-acoustic tilted transversely isotropic wave equations derived by setting the shear velocity along the axis of symmetry to zero have already been proposed (Zhou et al., 2006b; Fletcher et al., 2008; Zhang and Zhang, 2008).

We focus on a new pseudo-acoustic TTI wave equation derived using the asymptotically exact P-SV dispersion relation. For media with a constant vertical axis of symmetry and epsilon greater than or equal to delta, it is straightforward to implement modeling and migration with the vertical shear velocity set to zero. However, for modeling and migration in heterogeneous TTI media, a variably oriented axis of symmetry can cause instabilities with numerical implementations. Making the shear velocity along the axis of symmetry finite can remove propagation instabilities arising from tilt axis variation as well as from places where epsilon is less than delta. We will detail how to choose the shear velocity to stabilize propagation in heterogeneous TTI media, whilst minimizing SV waves considered as artifacts for acoustic modeling and migration.

Method

Solving the Christoffel equations for homogeneous TTI media gives three distinct wave modes: P, SV and SH (Tsvankin, 2001). The SH mode decouples, leaving a fourth-order dispersion equation describing propagation of the coupled P and SV modes. Correctly, this dispersion relation describes wavespeeds for the distinguished vector polarizations corresponding to the P or SV waves. However, we can also treat it as defining the propagation of a scalar, "pseudo-acoustic" wavefield. The exact P-SV TTI dispersion relation can be written as

$$\begin{aligned} \omega^4 = & \left[(v_{px}^2 + v_{sz}^2)(\hat{k}_x^2 + \hat{k}_y^2) + (v_{pz}^2 + v_{sz}^2)\hat{k}_z^2 \right] \omega^2 \\ & - v_{px}^2 v_{sz}^2 (\hat{k}_x^2 + \hat{k}_y^2)^2 - v_{pz}^2 v_{sz}^2 \hat{k}_z^4 \\ & + \left[v_{pz}^2 (v_{pn}^2 - v_{px}^2) - v_{sz}^2 (v_{pn}^2 + v_{pz}^2) \right] (\hat{k}_x^2 + \hat{k}_y^2) \hat{k}_z^2, \end{aligned} \quad (1)$$

where the hats over the wavenumbers (\hat{k}_x, \hat{k}_y and \hat{k}_z) indicate that they are evaluated in a rotated coordinate system aligned with the symmetry axis. Here ω is angular frequency; v_{pz} is P wave velocity in the direction normal to the symmetry plane; $v_{pn} = v_{pz} \sqrt{1+2\delta}$ is the P-wave normal moveout (NMO) velocity, again relative to the normal to the symmetry plane; $v_{px} = v_{pz} \sqrt{1+2\epsilon}$ is the P wave velocity in the symmetry plane; v_{sz} is the SV velocity normal to the symmetry plane; and ϵ and δ are anisotropic parameters defined by Thomsen (1986). The corresponding partial differential equation (PDE) in time, following immediately from equation 1, is cumbersome to solve as it is a fourth-order equation in time. It includes mixed spatial derivatives which require more computation than derivatives in a

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single spatial variable because differencing operators become two or three dimensional convolutions rather than one dimensional. For this reason, we often seek to find equivalent coupled lower-order systems.

We have analyzed the space of possible coupled second-order in time PDEs that can be derived from the P-SV dispersion relation in equation 1 (Fowler et al., 2008). Of the space of possible solutions the most efficient to implement is the family of coupled second-order PDEs parameterised with a non zero scalar α

$$\begin{aligned}\frac{\partial^2 p}{\partial t^2} &= v_{px}^2 H_2 p + \alpha v_{pz}^2 H_1 q + v_{sz}^2 H_1 (p - \alpha q) \\ \frac{\partial^2 q}{\partial t^2} &= \frac{v_{pn}^2}{\alpha} H_2 p + v_{pz}^2 H_1 q - v_{sz}^2 H_2 \left(\frac{1}{\alpha} p - q \right)\end{aligned}\quad (2)$$

where the differential operators H_1 and H_2 are given by

$$\begin{aligned}H_1 &= \sin^2 \theta \cos^2 \phi \frac{\partial^2}{\partial x^2} + \sin^2 \theta \sin^2 \phi \frac{\partial^2}{\partial y^2} + \cos^2 \theta \frac{\partial^2}{\partial z^2} \\ &+ \sin^2 \theta \sin 2\phi \frac{\partial^2}{\partial x \partial y} + \sin 2\theta \sin \phi \frac{\partial^2}{\partial y \partial z} + \sin 2\theta \cos \phi \frac{\partial^2}{\partial x \partial z} \\ H_2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - H_1,\end{aligned}\quad (3)$$

and θ is the dip measured to the vertical and ϕ the azimuth of the axis of symmetry. Note, that for VTI, the differential operator H_1 acts only in the vertical direction while H_2 acts in only the horizontal direction, and neither operator will contain any mixed space derivatives. In isotropic media, these two second-order PDEs in equation 2 are the same and wavefield p is equal to wavefield q . Whilst both the wavefields p and q are solutions to the fourth-order PDE corresponding to equation 1, we treat wavefield p as pressure and q as an auxiliary wavefield. The derivation of this family of wave equations can be found in the appendix. We implement equation 2 with a choice of $\alpha=1$.

Setting v_{sz} to zero

Starting from wave equation 2 and using Alkhalifah's trick of setting the shear velocity along the symmetry axis to be zero gives a pseudo-acoustic wave equation that can be more efficiently solved. When $\alpha=1$ equation 2 reduces in VTI media to the wave equation proposed by Du et al. (2008). When the symmetry axis is non-vertical it reduces to the wave equation proposed by Fletcher et al. (2008) and Zhang and Zhang (2008). When $\alpha=v_{pn}/v_{pz}$ equation 2 reduces in VTI media to the wave equation proposed by Duveneck et al. (2008). In this case, Duveneck et al. (2008) were able to physically interpret the p wavefield as the

horizontal stress component and the q wavefield as the vertical stress component.

We have found explicit centered finite-difference solutions to equation 2 when $v_{sz}=0$ can be unstable in some media with varying azimuth and dip angle. The instabilities usually start at locations in the azimuth and dip models where sharp contrasts exist. These instabilities appear to arise from the interaction of the SV-wave artifact with rapid variations in the tilt axis. Smoothing the model can stabilize wave propagation but it can significantly alter the kinematics of wave propagation. Our experiments indicated that adding finite v_{sz} terms provides an alternative and more accurate method to stabilize wave propagation. The question remains of what v_{sz} values to use. The following analysis details our approach to stabilizing propagation by using a reasonable finite v_{sz} in equation 2. Our approach also removes triplications from the SV wavefront and minimizes the anisotropic SV reflection coefficient.

Stabilizing propagation using a finite v_{sz}

The parameter

$$\sigma = \frac{v_{pz}^2}{v_{sz}^2} (\epsilon - \delta) \quad (4)$$

largely determines the kinematic signatures of SV-waves in TTI media (Tsvankin, 2001). Figure 1 displays the kinematics of P and SV wavefronts in a strongly anisotropic homogeneous VTI medium ($v_{pz}=3000\text{m/s}$, $\epsilon=0.24$, $\delta=0.1$) for various values of σ (and hence v_{sz}). As documented by Tsvankin (2001) triplications in the SV wavefront are removed for values of σ less than approximately 0.8. Experiments indicated that choosing v_{sz} large enough (and hence to ensure σ small enough) to remove SV wavefront triplications, gives stable wave propagation even with highly variable tilt axis orientation.

The reflection coefficient of SV-waves at a small-contrast interface between two weakly anisotropic TI media can be expressed as the sum of the corresponding coefficient in isotropic media $R_{iso}(\theta)$ and the anisotropic term

$$R_{aniso,SV}(\theta) \approx \frac{1}{2} (\sigma_1 - \sigma_2) \sin^2 \theta \quad (5)$$

where σ_1 and σ_2 are the parameter defined in equation 4 above and below the reflector (Tsvankin, 2001). Clearly if we choose v_{sz} to ensure a constant value of σ over the whole heterogeneous model, then this anisotropic term for the SV reflection coefficient will be zero everywhere. Taking this approach, in isotropic and elliptically anisotropic media we ensure that v_{sz} is set to zero.

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Numerical examples

In the following 2D numerical examples, wave propagation is simulated using high-order finite differencing to solve equation 2. To demonstrate our approach of stabilizing propagation using a finite shear velocity along the axis of symmetry, we use the BP 2D TTI model. Figure 2 displays a small region of this model. There is complex heterogeneity in all four parameters, including the dip angle for the axis of symmetry. Figure 3(b) displays a wavefield snapshot from a modeling experiment (without absorbing boundary conditions) in this part of the model. The modeling performed here set $v_{sz}=0$. Clearly the wave propagation is unstable where rapid variations exist in the symmetry axis dip direction. Figure 3(c) displays the same time wavefront snapshot as in Figure 3(b) but setting $v_{sz}=v_{pz}/2$. The wave propagation is now stable. However additional energy (highlighted on Figure 3(c)) is now present. This energy is caused by reflections of the SV artifact. Figure 3(d) again displays the same time wavefront snapshot but generated by setting v_{sz} to ensure $\sigma=0.75$ everywhere. The wave propagation is again stable but without the unwanted SV reflections. Figure 4 then displays the RTM image generated the same way for the complete BP synthetic model.

Conclusions

We propose a new stable wave equation for TTI media that can be solved as part of an acoustic anisotropic reverse-time migration algorithm using standard explicit finite-differencing. Forcing the shear velocity along the axis of symmetry in this equation to zero gives a simpler wave equation. We observed that explicit finite-difference solutions of this simpler wave equation can become unstable in transversely isotropic media which have rapid variations in the orientation of the axis of symmetry. We propose a solution to this problem using a small finite shear velocity along the axis of symmetry, chosen to remove triplications from the SV wavefront and minimize the anisotropic term of the SV reflection coefficient. The proposed solution works equally well for RTM in complex 3D TTI media.

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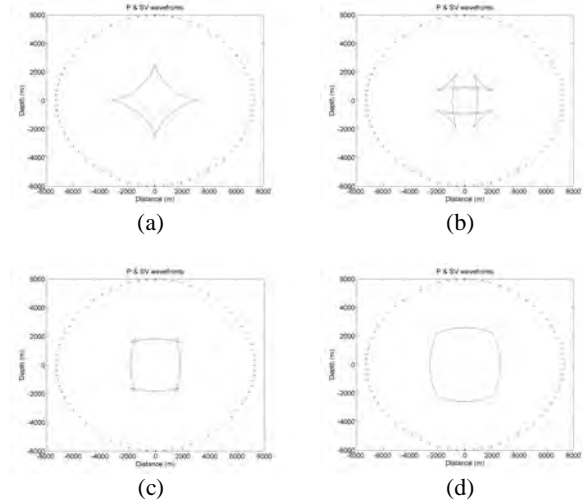


Figure 1. Kinematics of P-wave (crossed line) and SV-wave (solid line) in a homogeneous VTI medium: $v_{pz}=3000\text{m/s}$; $\varepsilon=0.24$ and $\delta=0.1$. The parameter $\sigma = v_{pz}^2(\varepsilon - \delta)/v_{sz}^2$ controls triplications in the SV wavefront: (a) $\sigma = \infty$; (b) $\sigma = 6.0$; (c) $\sigma = 1.5$; (d) $\sigma = 0.75$.

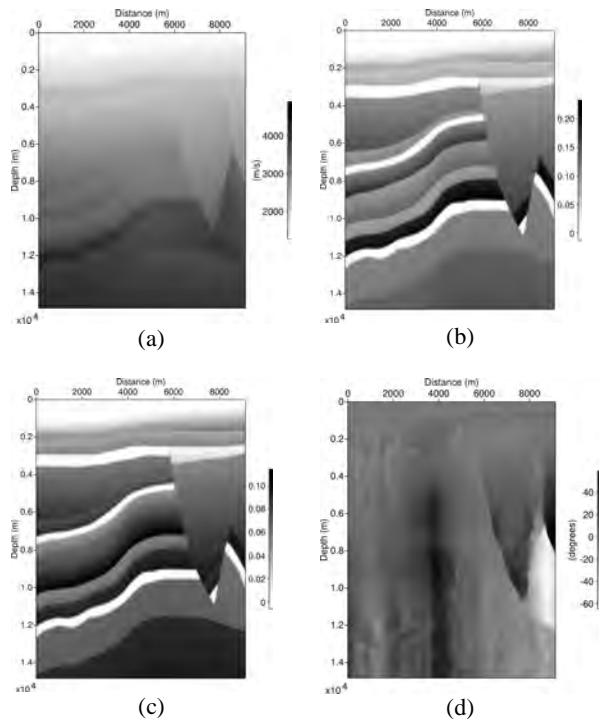


Figure 2. Partial BP TTI model: (a) v_{pz} ; (b) ε ; (c) δ ; (d) θ .

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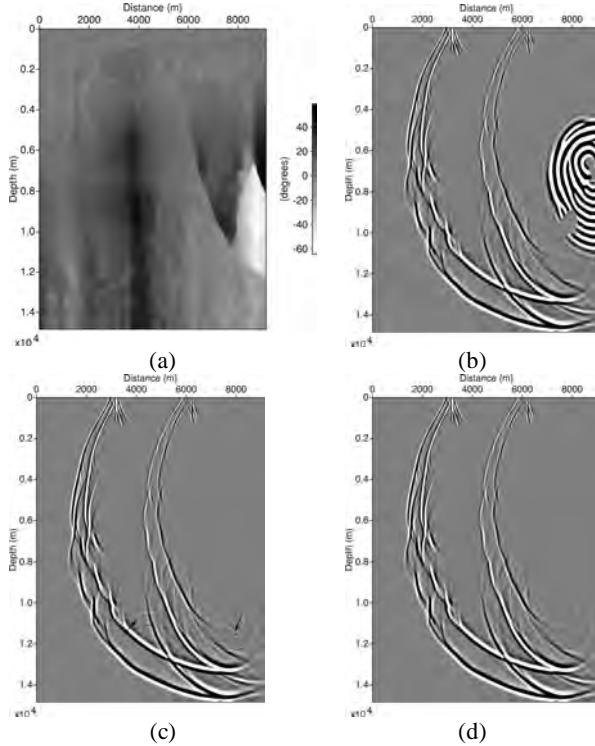


Figure 3. Wavefield snapshots in the BP 2D TTI model: (a) apertured model dip parameter θ ; (b) wavefield snapshot setting $v_{sz}=0$. The wave propagation is unstable where a high contrast is present in the dip field; (c) wavefield snapshot setting $v_{sz} = v_{pz}/2$. The propagation is stable but additional reflections are visible; (d) wavefield snapshot setting $\sigma=0.75$. Note that there are no absorbing boundary conditions employed in this experiment.

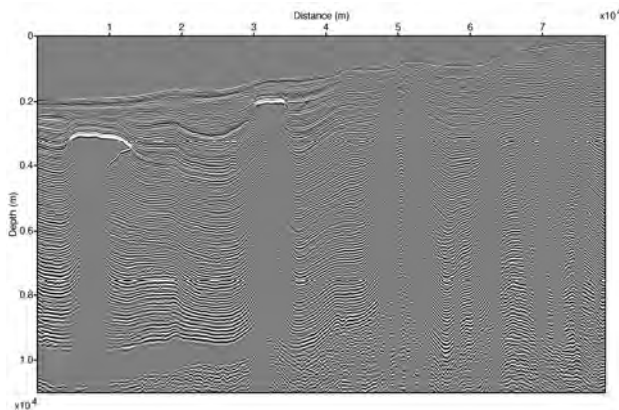


Figure 4. Prestack RTM image of the complete BP 2D TTI dataset.

Appendix

If we let θ and ϕ be the rotation angles (θ the dip angle measured to vertical, ϕ the azimuth), we then have for the rotated wavenumbers

$$\begin{aligned}\hat{k}_x &= k_x \cos \theta \cos \phi + k_y \cos \theta \sin \phi - k_z \sin \theta \\ \hat{k}_y &= -k_x \sin \phi + k_y \cos \phi \\ \hat{k}_z &= k_x \sin \theta \cos \phi + k_y \sin \theta \sin \phi + k_z \cos \theta.\end{aligned}\quad (\text{A-1})$$

The dispersion relation in equation 1 can then be rewritten as

$$\begin{aligned}\omega^4 &= ((v_{px}^2 + v_{sz}^2)f_2 + (v_{pz}^2 + v_{sz}^2)f_1)\omega^2 - v_{px}^2 v_{sz}^2 f_2 \cdot f_2 \\ &\quad - v_{pz}^2 v_{sz}^2 f_1 \cdot f_1 + (v_{pz}^2(v_{pn}^2 - v_{px}^2) - v_{sz}^2(v_{pn}^2 + v_{pz}^2))f_1 \cdot f_2,\end{aligned}\quad (\text{A-2})$$

where

$$\begin{aligned}f_1 &= k_x^2 \sin^2 \theta \cos^2 \phi + k_y^2 \sin^2 \theta \sin^2 \phi + k_z^2 \cos^2 \theta \\ &\quad + k_x k_y \sin^2 \theta \sin 2\phi + k_y k_z \sin 2\theta \sin \phi + k_x k_z \sin 2\theta \cos \phi \\ f_2 &= k_x^2 + k_y^2 + k_z^2 - f_1.\end{aligned}\quad (\text{A-3})$$

If we apply an inverse Fourier transform to these equations (using the relationships: $k_x \leftarrow -i \frac{\partial}{\partial x}$; $k_y \leftarrow -i \frac{\partial}{\partial y}$;

$k_z \leftarrow -i \frac{\partial}{\partial z}$; $\omega \leftarrow i \frac{\partial}{\partial t}$), we obtain the differential operators

$$\begin{aligned}H_1 &= \sin^2 \theta \cos^2 \phi \frac{\partial^2}{\partial x^2} + \sin^2 \theta \sin^2 \phi \frac{\partial^2}{\partial y^2} + \cos^2 \theta \frac{\partial^2}{\partial z^2} \\ &\quad + \sin^2 \theta \sin 2\phi \frac{\partial^2}{\partial x \partial y} + \sin 2\theta \sin \phi \frac{\partial^2}{\partial y \partial z} + \sin 2\theta \cos \phi \frac{\partial^2}{\partial x \partial z} \\ H_2 &= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - H_1.\end{aligned}\quad (\text{A-4})$$

Multiplying both sides of equation A-2 with the wavefield $p(\omega, k_x, k_y, k_z)$, considered to be the pressure wavefield, and introducing the new auxiliary function (parameterized by a non-zero scalar α)

$$q(\omega, k_x, k_y, k_z) = \frac{(v_{pn}^2 - v_{sz}^2)f_2}{\alpha(\omega^2 - v_{sz}^2 f_2 - v_{pz}^2 f_1)} p(\omega, k_x, k_y, k_z), \quad (\text{A-5})$$

equation A-2 can be written as

$$\begin{aligned}\omega^2 p(\omega, k_x, k_y, k_z) &= v_{px}^2 f_2 p(\omega, k_x, k_y, k_z) + v_{sz}^2 f_1 p(\omega, k_x, k_y, k_z) \\ &\quad + \alpha(v_{pz}^2 - v_{sz}^2) f_1 q(\omega, k_x, k_y, k_z).\end{aligned}\quad (\text{A-6})$$

Applying an inverse Fourier transform to both sides of the previous two equations gives equation 2. For seismic forward modeling, we must inject the source function in the right side of both equations.

EDITED REFERENCES

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