

## Anisotropic model building with uncertainty analysis

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### Summary

Velocity estimation is usually an ill-posed problem even for isotropic media. Widespread use of anisotropic imaging has been shown to aid better focusing and positioning. However, it greatly escalates the complexity of the model building and makes the velocity estimation much more ill-posed. Conventional techniques continue to rely on gradient-based methods that deliver a single solution (or realization) of the model to the user. Here we demonstrate an alternative approach that acknowledges the non-uniqueness of the problem. It delivers an entire suite of models that fit the data equally well, allowing the user to select the most geologically plausible solution.

### Introduction

In the past the goal of seismic imaging was to focus the data and provide a high quality subsurface image. In the last decade more emphasis has been placed on delivering a proper depth image that is as close as possible to the actual subsurface structure. To achieve this goal it is no longer enough to simply focus the data, but one has to use a realistic anisotropic depth model to perform such imaging.

It is well known that surface seismic data alone cannot uniquely resolve all the parameters of an anisotropic subsurface. In time imaging this is reflected in our ability to resolve NMO velocity and anellipticity but not the vertical velocity, and thus depth (Tsvankin, 2001). In depth imaging there is a more complicated relationship between the resolved parameters. We have more confidence in the imaging velocity and less confidence in the vertical velocity which prevents us from accurately predicting true vertical depths. What is less well appreciated is that we often cannot resolve all the parameters of the model even if we have well data to help constrain the vertical velocity. In a companion presentation Bakulin et al. (2009) showed a significant ambiguity between epsilon and delta in TTI and VTI for a case of joint tomography of common image gathers and vertical VSP traveltimes.

In this presentation we will use the methods described by Osypov et al. (2008) to characterize which combinations of parameters are resolved by a joint tomography experiment and which combinations of parameters are unresolved.

### Linear vs. Non-linear

The most general way to describe of the information in a particular combination of data is to define the likelihood function for that data. This is the probability of collecting the data we observe given a particular model. The likelihood is a potentially complicated and non-linear

function. If we assume Gaussian noise in the data we can characterize the likelihood using the misfit between the modeled and observed data (Taranatola, 2005). Figure 1 shows a hypothetical data misfit for a two parameter model. We can see that the misfit has multiple minima. We can also see a “trough” of good parameter combinations that all fit the data reasonably well. This trough does not lie along a straight line but is a banana shape.

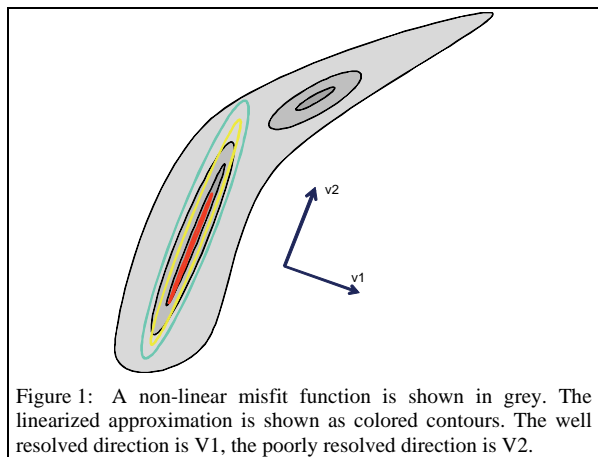


Figure 1: A non-linear misfit function is shown in grey. The linearized approximation is shown as colored contours. The well resolved direction is V1, the poorly resolved direction is V2.

If we wish to analyze the multi-dimensional misfit in the neighborhood of the minimum we often use a linearized approximation to the true non-linear problem. This gives a quadratic approximation to the misfit function which is indicated by the elliptical error contours shown in Figure 1. The linearized problem is much more mathematically tractable but we can see that it misses some of the features in the true model. The misfit function only has a single minimum and the trough of good solutions now lies along a straight line which is the long axis of the ellipse. Despite these limitations we will use the linearized approximation to explore the uncertainty in our inversion but we must always keep in mind that this approximation is less valid the further away we sample the model from our local minimum.

### Methodology

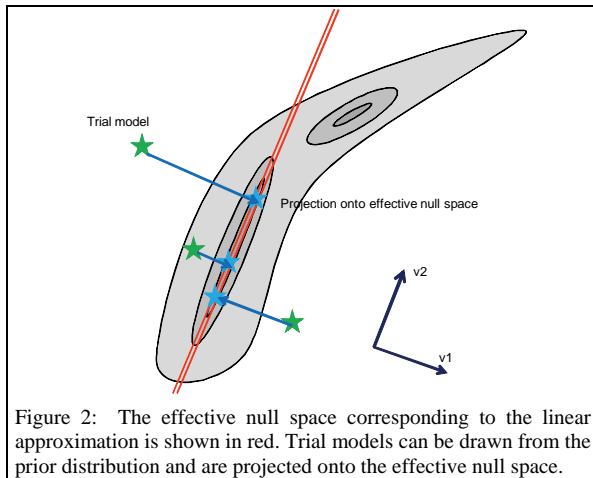
We can characterize the information in a joint surface and borehole tomography problem using a linearized approximation to the true problem. In this case the linearization involves the assumption that the rays do not change significantly when model is altered. The basic tomography problem is described by Woodward et al. (2008) and the analysis of uncertainty is described by Osypov et al. (2008).

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The linear operator that we analyze has three components:

- 1) The data covariance,  $C_d$ , describes the estimate of the errors in the data. In this case it handles the difference between the units of measurements of VSP traveltimes (seconds) and the units of non-flatness in common-image point gathers (meters).
- 2) The linear operator,  $A$ , contains the linear ray-based predictions of changes in measurements as the model changes.
- 3) The prior model covariance,  $C_m$ , describes the knowledge about the parameters that we had before we made the measurements.

The uncertainty in our models depends on all three components of the linearized tomography problem. The result of the uncertainty analysis is an eigen-decomposition of the posterior covariance. It defines combinations of model parameters that are well resolved by the data and which are poorly resolved.



In the 2D example shown earlier the best resolved direction is shown in Figure 1 by vector  $V1$ . This is associated with the largest eigenvalue in the decomposition. If we move away from the minimum in that direction the misfit function rises fastest. The worst resolved direction is shown by the vector  $V2$ . If we move in that direction the misfit function rises slowly, if at all. This direction is associated with the smallest eigenvalue in the decomposition.

If this eigenvalue were zero it would imply that the misfit does not change as we move in that direction. The misfit function would now be similar to the one shown in Figure 2. This combination of parameters would be in the “null space” of the operator. We can add any amount of that vector to our solution without changing the misfit. All those solutions would be equally valid, as far as the data misfit is

concerned. Remember however that this linearized approximation is not valid for huge changes in the model.

In models with more parameters this picture is harder to draw but the same principles apply. There are directions that are well resolved that correspond to large eigenvalues and poorly resolved directions that correspond to small or zero eigenvalues. A realistic model may have many thousands or millions of parameters. This means that it is infeasible to find all the eigenvectors. Our approach uses a partial eigen-decomposition of the problem that tells us the most important (best resolved) eigenvectors. The analysis can be stopped at a chosen minimum eigenvalue. All vectors that are not in the “well resolved” set are in the “effective null space” of the problem. Any change of model in that direction is predicted to have a change that is smaller than the cut-off eigenvalue.

In practice the effective null space can be sampled using the complement to the resolved space. If we have a trial solution drawn from the prior uncertainty we can find a null space solution by removing all components in the directions that are well resolved. In the 2D case this corresponds to removing the component of the trial model in direction  $V1$  which corresponds to projecting trial model onto the line defined by the local minimum and the direction  $V2$  (Figure 2). In the multi-dimensional case this is repeated for each resolved direction. Figure 2 shows the effective null space in red and the projections in blue.

This analysis is still limited by the linearization assumptions. Therefore, in order to study the global behavior of the non-linear objective function we must test our method using the true non-linear misfit to see whether we have stepped outside the bounds of the linear approximation.

### Synthetic example

Let us illustrate application of the uncertainty analysis using synthetic dataset described by Bakulin et al. (2009). Subsurface model is represented by horizontally layered TTI sediment with a uniform symmetry-axis tilt of 45 degrees. The model has smooth vertical variation of velocity and anisotropy (Figure 3). It has been proven that inversion of a combined dataset of narrow-azimuth surface seismic data and a dense vertical checkshot is non-unique in this case even though symmetry axis direction is assumed known and fixed. Whole series of equivalent TTI models fit the data. These models preserve the following combination of Thomsen parameters:  $V_{p0}(1 + 1.25\varepsilon - 0.75\delta) \approx V_{nmo}$  and  $\eta$ .

### Conventional workflow

In a real-world scenario model builder does not have information about non-uniqueness upfront. Our conventional workflow starts with an initial model and tomographic inversion drives it to a certain solution. In a

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TTI case at hand, we perform joint tomographic inversion of seismic and checkshot data using reflection tomography (Woodward et al., 2008) for three parameters ( $V_{p0}$ ,  $\varepsilon$  and  $\delta$ ) around the well. If we start with isotropic initial model then we recover model shown in Figure 3a. This model has negative Thomsen parameters that are not geologically plausible, but seismic and well data are matched. Without uncertainty analysis model builder is in a difficult circumstances. The only thing one can do with a conventional workflow (at a double the cost) is to repeat the process using new starting model. In our example with a new starting model having constant but positive values of anisotropy, tomography recovers a completely different solution (Figure 3b). At this point one can observe that both models have similar vertical velocity, similar  $V_{p0}(1 + 1.25\varepsilon - 0.75\delta) \approx V_{nmo}$  and  $\eta$ , but has no assurance as to whether this is a coincidence or a rule. One can conclude that problem is likely non-unique but conventional workflow does not provide guidance on how to proceed further, especially if all found solutions are not geologically plausible. Most likely model builder would decide to fix some parameters (say  $\delta$ ) and perform tomography for only two parameters. This however may lead to introducing assumptions inconsistent with the data (i.e. would be unable to flatten the gathers with them) or restrict the range of answers to only pre-determined  $\delta$  scenarios.

### New workflow with uncertainty analysis

Armed with the uncertainty analysis, one can pursue a different approach at no additional cost. For the sake of argument, we concentrate on uncertainty analysis performed around the first solution with negative Thomsen parameters (Figure 3a). For simplicity we pretend that our prior information allows us to have any value of Thomsen parameters larger than -0.3 and smaller than 0.3. As shown in Figure 2 we can sample our prior space and find its

projection onto the nullspace. In simple words each time we find a closest model to the prior model that fits the data. For our TTI example at hand this results in a 50 realizations shown in Figure 4 that carries a lot of information. First, it diagnoses to us numerically that we *have a non-uniqueness* problem which is an extremely powerful message. Second, it suggests that certain parameters combination, like  $V_{p0}(1 + 1.25\varepsilon - 0.75\delta) \approx V_{nmo}$  and  $\eta$  are much better constrained compared to  $V_{p0}$ ,  $\varepsilon$  and  $\delta$ . Let us pick three of these 50 realizations shown in red in Figure 4. They have a deviation of about 0.05 in  $\varepsilon$  and  $\delta$  which makes them meaningfully different from a retrieved solution in a practical sense. Figure 5 shows the seismic gathers after remigration and checkshot misfits after re-raytracing with these three realizations of new velocity model. We observe that gathers remain reasonably flat whereas checkshot misfit remains below 50 ms. Thus we confirm that all three realizations still fit the data within certain threshold of the misfit function (30,000). As a result of exploring the null-space, we obtain *range of plausible solutions* instead of a single one if problem is non-unique.

In our example we can not recover the second solution (Figure 3b) or true solution from the analysis done around the first one (Figure 3a). This is because they are too far apart in a model space, whereas the quadratic description of the misfit function is local as shown on cartoon Figure 2. Performing analysis around several solutions we can get a more complete description of the null-space. Alternatively, with a better prior information we can explore enough of a surroundings to arrive to a better choice. For example we expect that performing uncertainty analysis around the second solution with positive but incorrect Thomsen parameters (Figure 3b) we should be able to capture the true model as one of the nearby realizations or at least verify that the true model is plausible from the data standpoint without repeating the inversion.

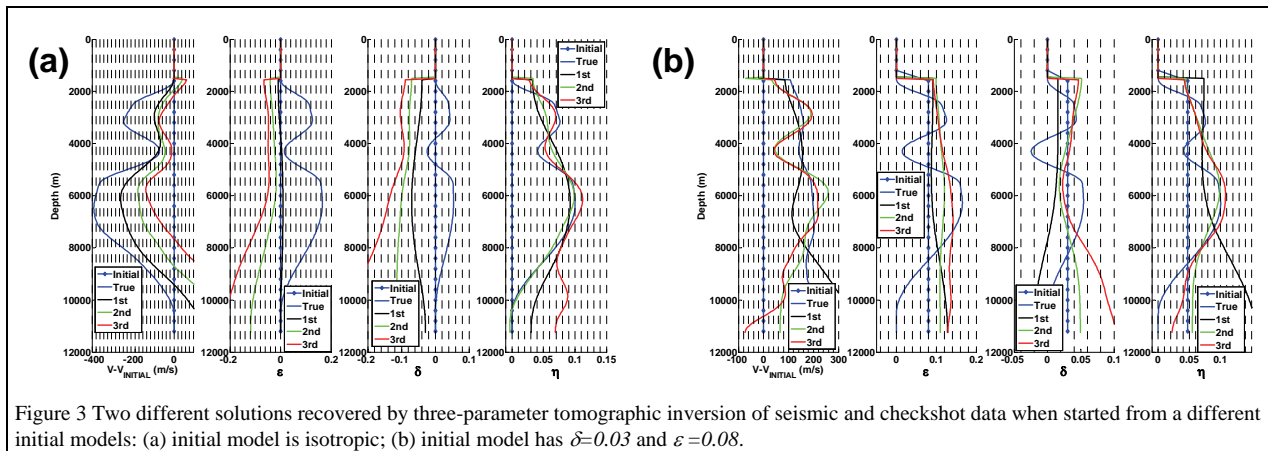


Figure 3 Two different solutions recovered by three-parameter tomographic inversion of seismic and checkshot data when started from a different initial models: (a) initial model is isotropic; (b) initial model has  $\delta = -0.03$  and  $\varepsilon = 0.08$ .

## Conclusions

Leaving all mathematical complexity of the null-space in the background, from the practical standpoint, the proposed methodology enables a novel approach to the model building. The null-space analysis allows to reveal the non-uniqueness of the anisotropic inversion as well as to explore range of equivalent models providing similar fit to the data. Without null-space analysis model builder is stuck with a solution and no other information as to how well it

is resolved and whether it is the only one. With the uncertainty analysis, model builder has a far better alternative. Firstly, he can detect the non-uniqueness. Secondly, he can explore range of equivalent models and pick one that he considers more geologically plausible based on any a priori information. Alternatively, null-space analysis can reveal what additional data are required to constrain the model.

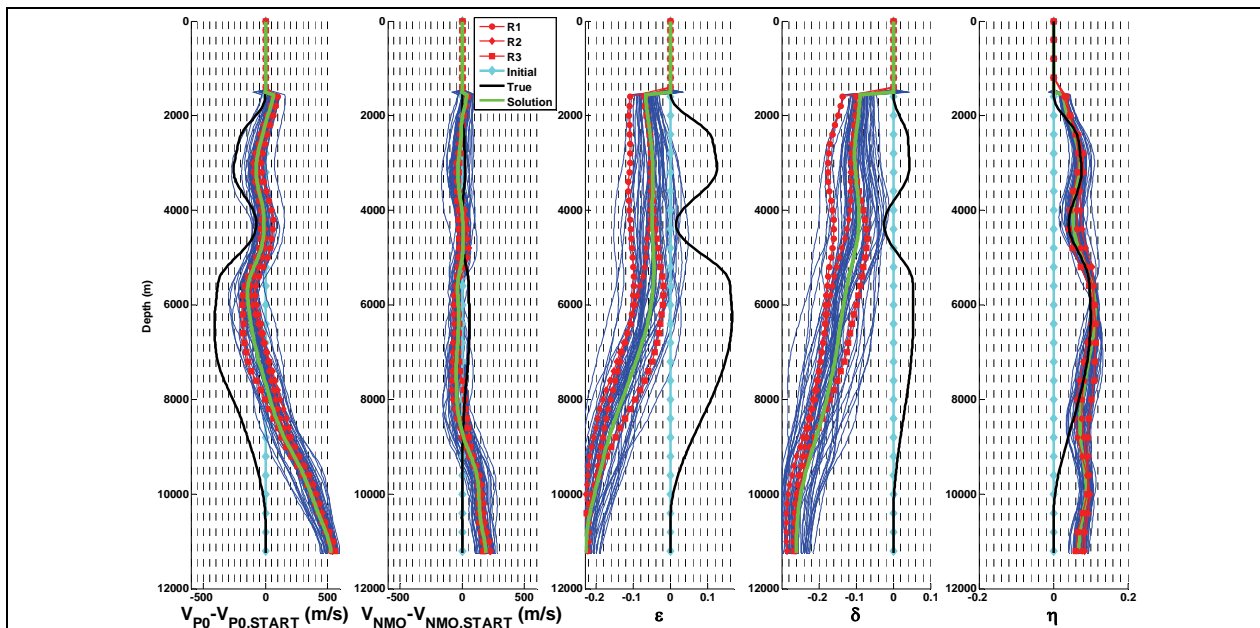


Figure 4. Fifty null-space realizations (thin blue lines) obtained by uncertainty analysis around the first solution from Figure 3a. Three realizations highlighted in red are selected for further testing. Observe tight distribution of  $\eta$  and  $V_{P0}(1 + 1.25\varepsilon - 0.75\delta) \approx V_{nmo}$  whereas  $V_{P0}$ ,  $\varepsilon$  and  $\delta$  individually are not well constrained.

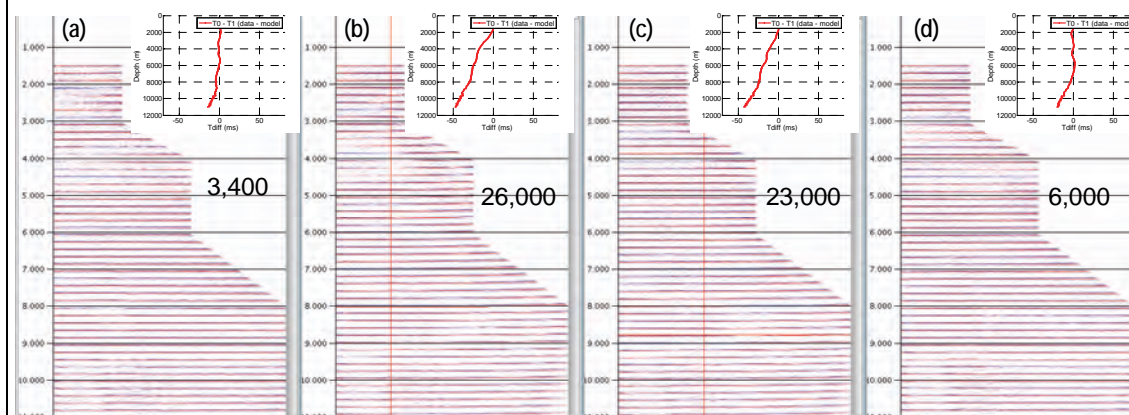


Figure 5: Image gathers for various models together with the inserts showing checkshot misfits. Number on each plot identifies total value of misfit function characterizing unflatness of the gathers as well as checkshot misfit: (a) found solution; (b)-(d) realizations 1, 2, and 3 from the nullspace (Figure 4). Note that gathers remain rather flat for all models, whereas checkshot residual remains less than 50 ms.

### **EDITED REFERENCES**

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