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Seismic Full-waveform Inversion of Salt Geometry Using a Level Set Approach

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SUMMARY

An accurate definition of geometry of complex subsurface bodies, such as salt intrusions, is crucial for imaging below such targets. Full-waveform inversion (FWI) is a method for building high-resolution seismic velocity models from nonlinear iterative minimization of the misfit between observed and synthetic seismic data. The ability of FWI to accurately recover the geometry of salt bodies can be limited by several factors, with the lack of low frequencies in the data being one of them, thus, requiring the salt geometry to be often manually interpreted. In this paper we modify the FWI algorithm to use a level set representation to parameterize and invert for the geometry of salt bodies, without adding to computational cost of FWI.
Introduction

Due to recent advances in the availability of computing resources, seismic full-waveform inversion (FWI) (Tarantola, 1984) has gained much interest because of its ability to produce high-resolution earth models (Virieux and Operto, 2009). However, the task of accurately recovering the geometry of salt bodies, typically found in geologically complex marine environments like the Gulf of Mexico, still presents a great challenge to FWI. Therefore, the salt geometry often must be manually picked by seismic interpreters, which is not only subjective but also a time-consuming and costly process. Better definition of the salt geometry has been shown to greatly improve imaging in the subsalt sedimentary regions, as shown by Vigh et al. (2010). In this paper, we present an approach to directly invert the geometry of the salt bodies, using FWI, by partitioning the inversion domain into salt and sediment regions and using a level set representation to parameterize the salt geometry.

The level set method was first developed by Osher and Sethian (1988) to describe the motion of curves and surfaces. The implicit parameterization of the level set method has many advantages over explicit parameterization of the geometry using polygonal meshes (Zhang et al., 2007; Li et al., 2010). The implicit approach is able to handle sharp corners and cusps in the geometry, as well as making topological changes like merging and separation of bodies very easy, besides having a straightforward extension to three-dimensional geometries.

The use of level set representation in the context of solving inverse problems was introduced by Santosa (1996). Since then, the level set approach has been used frequently for shape optimization in various inverse problems (Osher and Santosa, 2001; Burger, 2001, 2002). In solving geophysical inverse problems, the same approach was used by Dorn et al. (2000, 2001) to reconstruct the shapes of objects in electromagnetic tomography. Here, we adapt the level set approach to solve for the salt geometry such that it requires minimal changes to the FWI algorithm and does not add to its computational expense.

Method

The classical least-squares formulation of the full-waveform inversion problem can be described as follows. We have $F : M \rightarrow \mathbb{R}^m$ as the forward modeling operator that maps the property model $x \in M \subset \mathbb{R}^n$ (e.g., $v_p$, $\rho$, etc) defined over the inversion domain $D$ using a gridded representation, to the data domain, and $d \in \mathbb{R}^m$ is the observed data. We now solve the following optimization problem,

$$\min f(x) = \frac{1}{2} \| F(x) - d \|_2^2$$

where $\| \cdot \|_2^2$ represents the squared L2 norm. At each iteration $k$, the model is updated according to rule $x_{k+1} = x_k + \alpha_k p_k$, where $\alpha_k$ is the step length determined by a line search procedure, and the direction $p_k$, could be, depending on the optimization technique being used, one of steepest descent, conjugate gradient, or Newton/quasi-Newton directions.

Level Set Approach

For any real valued function $f : \mathbb{R}^k \rightarrow \mathbb{R}$, a level set is the set where the function takes on a given constant value, $\Gamma(\alpha) = \{ x \in \mathbb{R}^k : f(x) = \alpha \}$. The level set representation involves constructing a function $\phi : \mathbb{R}^2 \rightarrow \mathbb{R}$ (in 2D) such that its zero level set, $\Gamma(0)$, is the boundary $\partial \Omega$, of the domain $\Omega$ that we want to represent and invert for (Figure 1). The level set function $\phi$ is set such that it is positive in $\Omega$ (salt) and negative in $\Omega^c$ (sediment). Using the level set representation to parameterize the salt geometry, we reformulate the inversion as follows.

Let $\hat{x}$ be the implicit model defined using a level set function $\phi \in D$, and $g : D \rightarrow M$ be the operator that maps the implicit model onto a grid. The new objective function is then given by,

$$\min \hat{f}(\hat{x}) = \frac{1}{2} \| F(g(\hat{x})) - d \|_2^2$$

(2)
The gradient of the new objective function is computed using a simple application of the chain rule,

\[ \nabla \hat{f}(\hat{x}) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \nabla g_i(\hat{x}) \quad (3) \]

where \( g_i(\hat{x}) = x_i \).

**Level Set Evolution**

At each iteration, the perturbations to the level set boundary are made by evolving the level set function \( \phi \). The equation of motion controlling the evolution of the level set function is given by,

\[ \phi_t + V \cdot \nabla \phi = c \kappa |\nabla \phi| \quad (4) \]

where, \( \kappa = \nabla \cdot (\nabla \phi / |\nabla \phi|) \) is the curvature of the surface, \( c \geq 0 \) is the curvature weight, and \( V \) is the directional force acting on the level set surface. By appropriately choosing the directional force \( V \) and solving the above differential equation, we can control the perturbations that are made to the level set surface representing the salt geometry.

In accordance with the steepest descent method, we use the negative of the gradient as the force acting on the level set surface to evolve \( \phi \). That is, at iteration \( k \), evolve \( \phi \) according to

\[ \phi_k - \nabla \hat{f}_k(\hat{x}_k) \cdot \nabla \phi = c \kappa |\nabla \phi| \quad (5) \]

The evolution is stopped when \( \alpha_k = t_k - t_{k-1} \), the step length along the time dimension, meets certain conditions (Gill et al., 1981) determined by a line search procedure. The line search ensures that the value of the objective function is reduced at every iteration. At the end of the iteration, the gradient is recomputed using the new solution for \( \phi \) and the method is restarted using the newly computed gradient.

**Extrapolation**

Because the gradient \( \nabla \hat{f}_k(\hat{x}) \) is defined only on \( \partial \Omega \), to be able to use equation 5 to evolve the level set function, we must provide a continuous extension of the gradient off \( \partial \Omega \) onto \( D \). There are several ways in which this can be done, and we choose the method outlined by Fedkiw et al. (1999). First, we choose the level set function to be the signed distance function (Osher and Fedkiw, 2003),

\[ \phi(x) = \inf_{y \in \partial \Omega} ||x-y||_2 S(\phi_0) \quad (6) \]

where \( S(\phi) = \frac{\phi}{\sqrt{\phi^2+\delta}} \) and

\[ \phi_0(x) = \begin{cases} 1 & x \in \Omega \\ -1 & x \in \Omega^c \end{cases} \]

A constant extrapolation of the gradient along the normal direction to \( \partial \Omega \) is then computed by solving the following advection equation,

\[ I_t \pm \frac{\nabla \phi}{|\nabla \phi|} \cdot \nabla I = 0 \quad (7) \]
where $I$ is the quantity being advected, which in our case, will be the components of the gradient vector. Because, at every iteration, we can assume that the perturbation to the boundary is small, this extrapolation step can be optimized such that the gradient must be defined only in a small neighborhood of the boundary $\partial \Omega$.

**Results**

We tested our method on a new 2D model we created as shown in Figure 2a. This model has a linearly increasing velocity in the sediment region and has two salt bodies with geometries derived from the subsalt multiples attenuation and reduction technology (SMAART) Pluto 1.5 model. The velocity model is defined on a 5,000 (in X) by 800 (in depth) grid with a grid interval of 25 ft in both dimensions. The model has a constant velocity (4,290 ft/s) water layer at the top up to a depth of 2,300 ft. The salt velocity is chosen to be constant at 14,800 ft/s. The observed data were synthetically generated using an acoustic propagator with true velocity and constant density. A fixed-spread acquisition geometry was used with a source interval of 75 ft and a receiver interval of 25 ft with a record length of 8 s. The source depth was same as the receiver depth at 25 ft.

To generate the starting model for the inversion, the entire salt boundary was shrunk by a normal force to the surface. The background sediment velocity was assumed to be known and kept unchanged throughout the inversion. Figure 3a shows the difference of the true velocity and the starting velocity for the inversion. The magnitude will be zero everywhere except in the neighborhood of the salt boundary. The inversion using the new level set approach was run starting with the shrunk salt model with a 3-Hz low-pass filter applied to both the predicted and observed data. The objective function (log of data misfit) monotonically decreases along the iterations as the inversion progresses, suggesting convergence as shown in Figure 2b. The converged solution can be seen in Figure 3b, which shows the difference of the true velocity and the inverted velocity after 18 iterations, and in Figures 4a and 4b, which show the velocity profiles at $x=2100$ (52,500 ft) and $x=3500$ (87,500 ft), respectively.

![Figure 2](image)

*Figure 2* (a) True velocity. (b) Plot of log of data misfit versus iterations.

**Conclusions**

We showed that our adaptation of the level set approach within the FWI workflow can potentially invert for salt geometries, and can, therefore, be used to construct an accurate definition of the salt geometry, which is very crucial for imaging subsalt targets. We are currently investigating the extent of our method on more complicated models.

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**Figure 3** (a) Difference plot of true velocity and starting velocity. (b) Difference plot of true velocity and inverted velocity.

**Figure 4** Velocity depth profiles with true velocity in red, starting velocity in blue, and inverted velocity in black at (a) x=2100 (52,500 ft) (b) x=3500 (87,500 ft).

**References**