Th N116 04

Optimizing the Use of Gradient Measurements in Wavefield Reconstruction - A Bayesian Noise Tracking Approach

Y.I. Kamil* (Schlumberger), P. Loganathan (Schlumberger), M. Vassallo (Schlumberger), M. Cowman (Schlumberger) & A. Raskopin (Schlumberger)

SUMMARY

The second order statistics of the noise seen in the recent continuous line acquisition data from multimeasurement streamers vary in frequency, time and space domains. Incorrect estimation of such noise statistics can lead to poor reconstruction of the wavefield by the generalized matching pursuit (GMP). A new noise power estimation technique based on a Bayesian recursive framework is proposed. This algorithm can not only update the noise power estimation when the signal absence is presumed, but also track dynamically to the realistic levels of non-stationary noise. Results using a 3D test data shows improved noise estimation performance over the original method that assumes the noise statistics is stationary within any shot gather. This improvement is achieved by reducing the leaked turn down noise into the GMP output.
Introduction

Generalized matching pursuit (GMP) for joint interpolation and deghosting of multimeasurement streamer data was introduced by Özdemir et al. (2010). GMP is applied to individual shot records and utilizes the combination of pressure ($P$) and its crossline and vertical gradient measurements (denoted by $P_V$ and $P_Z$, respectively) to generate, a spatially well-sampled dataset in all directions, upgoing pressure wavefield. In GMP, the unknown upgoing pressure wavefield is modelled as a summation of monochromatic basis functions. The basis functions that account for 3D receiver ghost effects are selected by iteratively matching the measurements. GMP creates a cost function that merges contributions from measurements of a different nature, specifically pressure and acceleration. These measurements are affected by different types of noise; the noise measured by accelerometers can be several orders of magnitude stronger than the noise measured by hydrophones, especially at low frequencies (Özdemir et al. (2011)). Different standard noise attenuation approaches are applied to attenuate the noise in the multimeasurement data before the application of GMP. Nevertheless, GMP needs to address the different characteristics of the residual noise on the data.

Vassallo et al. (2011) demonstrated that by using adaptive weights for the crossline and vertical gradient measurements in the GMP cost function, the impact of noise can be minimized in the reconstructed upgoing wavefield. The weights must take into account the signal-to-noise ratio (SNR) of the different measurements to minimize any noise leakage and avoid potential reduction in the quality of the reconstructed upgoing wavefield. When the SNR is low in a measurement, its contribution is weighted down. As a result, the overall cost function gets affected by reliable measurements only. Therefore, for optimal performance, the SNR needs to be estimated in time, space and frequency. If the noise is stationary, SNR can be easily estimated from space-time windows where the noise dominates. Generally the noise is not stationary (i.e., its statistics changes with offset and/or time), and the SNR needs to be tracked in space and time. The method presented here is based on a Bayesian recursive framework that tracks information about the second order statistics of the noise in the multimeasurement data.

GMP and Nonstationary Noise Mitigation

Reconstruction of the upgoing pressure wavefield from the coarsely sampled pressure, crossline and vertical component of pressure gradient, $P$, $P_V$ and $P_Z$ is an under-determined inverse problem. GMP addresses this problem by iteratively solving the cost function $M$ at the $l$th iteration as a function of the complex amplitude $A_l$ and crossline wavenumber $k_{y,l}$:

$$M(A_l, k_{y,1}) = \sum_{n} |r_{y}^{l}(y_n, A_l, k_{y,l})|^2 + \lambda_{V} |r_{P_V}^{l}(y_n, A_l, k_{y,l})|^2 + \lambda_{Z} |r_{P_Z}^{l}(y_n, A_l, k_{y,l})|^2,$$

(1)

where $r_{y}^{l}$, $r_{P_V}^{l}$ and $r_{P_Z}^{l}$ are the residuals corresponding to the $P$, $P_V$ and $P_Z$ measurements respectively, and $y_n$ is the position of the $n^{th}$ crossline sample. The $\lambda_{V}$ and $\lambda_{Z}$ are the adaptive weights used to avoid the reduction in the quality of the reconstructed upgoing wavefield when the vertical and/or the crossline gradient measurements are noisy. These weights are computed as a function of the SNR on the gradients relative to the SNR on the pressure data.

Because the seismic signal energy changes with time ($t$), space ($x$) and frequency ($f$), the weights $\lambda_{V}$ and $\lambda_{Z}$ vary according to local estimates of $\text{SNR}(f,x,t)$. If the noise is assumed to be stationary within a shot gather, one way to estimate the noise power spectrum is by averaging the data power spectrum from space-time windows where the noise dominates. Therefore, to compute the local SNR, input traces are split into space-time windows. The SNR is estimated for each window by comparing the data power spectrum to the noise power spectrum estimate. Generally, the noise power spectrum varies in space and time and is not stationary. Estimation of the noise power spectrum from windows dominated by noise can cause overestimation or underestimation of the local SNR in windows where the noise behaves differently. If the weights are overestimated, this can lead to an increase in the noise leakage in the reconstructed GMP output. If the weights are underestimated, this may result in inaccurate reconstruction and deghosting. The robustness of the SNR estimation, requires the ability to estimate and track variations in the statistics of the noise model. Moreover, instead of estimating
the spectrum in each individual measurement independently, a multimeasurement noise power spectrum tracking approach is required to produce more accurate results. We propose a multimeasurement noise power spectrum tracking method within a Bayesian framework.

The main advantage of the Bayesian framework is that it combines the information in the multimeasurement data with any prior knowledge (Kay (1998)). It can incorporate information from multiple origins, all with varying degrees of uncertainty. Within the Bayesian framework, different prior information can be used to track the variations in the noise statistical model in space and time. For example, if the noise power spectral density can be assumed to be slowly varying in time, initial prior information can be used to track the variations in the noise statistical model in space and time.

To evaluate (2), the SPP needs to be evaluated by employing the Bayesian theorem:

\[
\Pr \left( H_1 \mid X(m) \right) = \frac{\Pr(H_1) \cdot f_{X(m)\mid H_1}(X(m))}{\Pr(H_1) \cdot f_{X(m)\mid H_1}(X(m)) + \Pr(H_0) \cdot f_{X(m)\mid H_0}(X(m))},
\]

where \( H_1 \) indicates signal presence and \( \Pr \left( H_1 \mid X(m) \right) \) is the conditional signal presence probability (SPP). Note that in the absence of the signal (\( H_0 \)), the noise PSD simplifies to the PSD of the input data as shown in the first expression of Eq.(2). In the presence of signal, the noise PSD can be estimated by smoothing the estimated noise power spectrum, by assuming certain degrees of correlation between noise power in these dimensions. While the formulation in Eq.(2) includes all measurements, it can be easily simplified to the subsets of the multimeasurement to denote the correlation between noise power in these dimensions. While the formulation in Eq.(2) includes all measurements, it can be easily simplified to the subsets of the multimeasurement to denote the presence in multimeasurement data can be derived based on high SNR measurements.

To estimate the SNR, the multimeasurement data are divided into overlapping space-time windows and transformed into the frequency domain. Let \( X(m) \) denote the complex spectral vector of the noisy multimeasurement vector \([P(m) \ P_r(m) \ P_s(m)]^T\), where \( m \) represents the time window index. For notational simplicity, we omit the dependency on frequency and offset. Let \( \mathbf{N}(m) \) be the additive noise that is assumed to be independent from the signal. For a time window, \( m \), the noise power spectral density (PSD) is defined by \( E\{ \mathbf{N}(m) \mathbf{N}^H(m) \mid X(m) \} \). Under signal presence uncertainty, the noise PSD given the data is expressed as,

\[
E\{ \mathbf{N}(m) \mathbf{N}^H(m) \mid X(m) \} = \begin{cases} 
1 - \Pr \left( H_1 \mid X(m) \right) & E\{X(m)X^H(m)\} + \\
\Pr \left( H_1 \mid X(m) \right) & E\{\mathbf{N}(m) \mathbf{N}^H(m) \mid X(m), H_1 \}, 
\end{cases}
\]

where \( H_1 \) indicates signal presence and \( \Pr \left( H_1 \mid X(m) \right) \) is the conditional signal presence probability (SPP). Note that in the absence of the signal (\( H_0 \)), the noise PSD simplifies to the PSD of the input data as shown in the first expression of Eq.(2). In the presence of signal, the noise PSD can be estimated by smoothing the estimated noise power spectrum, by assuming certain degrees of correlation between noise power in these dimensions. While the formulation in Eq.(2) includes all measurements, it can be easily simplified to the subsets of the multimeasurement to denote the corresponding single measurement. After estimating the raw noise PSD, the noise PSD estimation, \( \tilde{\mathbf{R}}_{NN}(m) \), can be recursively smoothed to reflect slow varying nature of the noise statistics in time as follows:

\[
\tilde{\mathbf{R}}_{NN}(m) = \mu \tilde{\mathbf{R}}_{NN}(m-1) + (1 - \mu)E\{\mathbf{N}(m) \mathbf{N}^H(m) \mid X(m)\},
\]

with \( 0 \ll \mu < 1 \), controlling the contribution of the previous and the current noise estimation.

To evaluate (2), the SPP needs to be evaluated by employing the Bayesian theorem:

\[
\Pr \left( H_1 \mid X(m) \right) = \frac{\Pr(H_1) \cdot f_{X(m)\mid H_1}(X(m))}{\Pr(H_1) \cdot f_{X(m)\mid H_1}(X(m)) + \Pr(H_0) \cdot f_{X(m)\mid H_0}(X(m))},
\]

where \( \Pr(H_1) \) and \( \Pr(H_0) \) are the signal presence and absence prior probabilities, respectively. \( f_{X(m)\mid H_1} \) and \( f_{X(m)\mid H_0} \) are the likelihood functions of the data in the case of signal presence and absence, respectively. Note that the prior probabilities can take into account other information such as the time and the offset of the data, the depth of the sea floor, seismic interference and etc. To evaluate the likelihood functions, we assume complex Gaussian distribution of the signal and noise complex spectral coefficients as follows:

\[
\begin{align*}
\tilde{f}_{X(m)\mid H_1}(X(m)) &= \frac{1}{\pi^d \det(\tilde{\mathbf{R}}_{NN,prior})} \exp \left( - \mathbf{X}(m)^H \tilde{\mathbf{R}}_{NN,prior}^{-1} \mathbf{X}(m) \right) \\
\tilde{f}_{X(m)\mid H_0}(X(m)) &= \frac{1}{\pi^d \det(\tilde{\mathbf{R}}_{XX,prior})} \exp \left( - \mathbf{X}(m)^H \tilde{\mathbf{R}}_{XX,prior}^{-1} \mathbf{X}(m) \right)
\end{align*}
\]
where $K$ is the multimeasurement dimension of the vector $X(m)$ (i.e., $K=3$, if $X(m)=[P \ P_Y \ P_Z]^T$), and the estimated matrices $\hat{R}_{NN,prior}$ and $\hat{R}_{XX,prior}$ are the prior noise and the measurement covariance matrices, respectively. This prior information can be obtained from different sources. For $\hat{R}_{NN,prior}$ for example, assuming a certain degree of correlation between the noise covariance matrices present in neighbouring windows, it is reasonable to use the averaged spectral noise power estimation of the previous windows. The signal prior covariance matrix $\hat{R}_{XX,prior}$, however, usually exhibits a larger degree of fluctuations between successive time windows. Therefore, previous estimate of the signal covariance cannot be used. Alternatively, if the SNR in one measurement is known to be very high, that measurement can provide a reliable estimate of the signal power, and the prior signal covariance matrix can then be derived using the prior knowledge of ghost operators to map the signal from one measurement to the other.

**Results**

A 3D test data set was acquired with a multimeasurement streamer in the Barents Sea using a continuous line acquisition (CLA) (Patenall and Brice 2014, Mahat et al. 2015). As expected, elevated noise levels were observed on the accelerometer measurements during the turns. Therefore, after employing the standard noise attenuation techniques, additional noise attenuation was applied to remove remnant turning noise from the accelerometers. Figure 1 shows the input multimeasurement data after the standard noise attenuation process at cable 1 (outer streamer) for a single shot from the turning section. The residual noise impacts on the accelerometers is clearly visible, marked using the red arrows in the figure. It can be also noticed that the noise is mostly nonstationary. We use the original method of estimating the weights in GMP, which assumes the noise within a shot gather is stationary, to compare against the results using the new proposed method that can deal with nonstationary noise.

Figure 2 shows an inline view of the reconstructed total pressure at the original (cable 1) and virtual cable (cable 1.5) positions. Panels a and b show the total reconstructed pressure result using the original noise model estimation. It can be observed that the noise leakage is minimal at original cable position, however noise leakage is visible at the virtual cable (as indicated by red arrows). Panels c and d show the total reconstructed pressure result using the proposed noise model estimation technique. Since the proposed method can account for the nonstationary behaviour of the noise, the noise leakage is less visible.

![Figure 1 Inline view of the input (a) pressure (b) crossline acceleration (c) vertical acceleration measurement to GMP at cable 1. Red arrows indicate areas of residual turn noise after standard noise attenuation techniques.](image-url)
Conclusions

We present a promising technique for reliably tracking the noise power spectrum and hence the SNR in space and time. The technique is based on a Bayesian recursive framework that tracks information about the second order statistics of the noise. The main advantage of the Bayesian framework is that it translates the information brought by the multimeasurement data as prior knowledge into the posterior distribution. This allows further processing techniques to take into account of the actual SNR as a function of frequency, time and space. We applied our proposed SNR estimation approach to the case of joint interpolation and 3D deghosting of multi-measurement data via GMP. The new SNR estimation method has allowed GMP to use the information in the gradient measurements effectively while providing a robust shield against residual turn noise on gradient measurements that could have otherwise leaked into the output pressure wavefield.

Acknowledgments

We would like to thank Peter Vermeer, Philippe Caprioli, Wouter Brouwer, Chris Cunnell and Pete Watterson for useful discussions. We thank Schlumberger for permission to publish this work.

References


Mahat, S., Ramsamair, R., Cunnell, C. and Watterson, P. [2015] Increasing acquisition efficiency by acquisition of data during turns using a multimeasurement streamer, 77th EAGE Conference and Exhibition.

