Geophysical modeling through simultaneous Joint Inversion of Seismic, Gravity and Magnetotelluric data

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SUMMARY

In this paper we present our recent developments in the field of Joint Inversion of different geophysical parameters. Our work was aimed at the improvement of seismic imaging through a better velocity model estimate, integrating geological considerations within a data-driven procedure. Our tests were carried out for Seismic/Gravity and Seismic/Magnetotelluric (MT) simultaneous Joint Inversion. Results show the possibility of a successful application to real data.

INTRODUCTION

Effective depth imaging through migration can be achieved only if a precise estimate of velocity model is available. It is known that a wrong velocity model can cause severe mispositioning of seismic events during migration.

Derivation of a reliable velocity model can be performed through different approaches. The so called model-driven methods, transform a geological section directly into a velocity model. These methods not always are able to provide a velocity model that agrees with measured data (i.e. arrival times).

On the other side there are data-driven methods. These, following a more rigorous approach (minimization of a cost function), yield always to fit measured data but, often, the final velocity structure might not agree with geological considerations.

The best model building workflow, should be the integration of data-driven and model-driven approaches. However, the inversion of geophysical parameters still remains an ill posed problem, not fully soluble through regularization. In general, more reliable estimates can be achieved integrating different sources of information (seismic arrival times, gravity measurements, magnetotellurics, interpretation, etc.).

In the past, Dell’Aversana et al. (2002) (see also Dell’Aversana (2003)), have performed integration of different-domain geophysical data, deriving, first, a model in one domain (generally velocity), transforming it to the other domain (through some estimated empirical functions) and performing further inversion into the other domain. Finally, the resulting model was back-transformed into the original domain, to improve depth imaging. Several difficulties are involved in the actual implementation of this approach: one of these seems to be the fact that, inversion carried out into the second domain, can bring inversed parameters out of the range for which the empirical function for back-transformation is defined. Therefore, a model that initially agreed with data into the first domain and that, after transformation and further inversion, also agreed with data in the second domain, can be no more in agreement with initial data after back-transformation. Furthermore, this approach gives more importance to the first domain over the second one (because the starting model comes from the first domain), with the harmful effect of possible propagation of model errors into the transformed geophysical domain.

In this paper, we present a workflow where seismic data residuals, gravity data, magnetotelluric (MT) data, geophysical constraints and interpretation can be quantitatively integrated into a simultaneous Joint Inversion (JI) procedure, able to estimate multi-parametric models. Figure 1 qualitatively shows these concepts. The application of the method to synthetic datasets is discussed in the present paper.

JOINT INVERSION FORMULATION

A brief description of the formulation of the Joint Inversion problem in the restricted case of the Joint Inversion of only two domains is provided below. The same approach can be extended to multiple domains.


All inverse problems try to find the set of parameters that minimize an objective function. We define an expanded model vector

\[
m = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}
\]  
(1)

where \(m_1\) and \(m_2\) are, respectively, models of the two different domains (e.g. seismic velocity and density or seismic velocity and resistivity). The objective function for Joint Inversion, with a formulation that follows from theory explained in Tarantola and Valette (1982) and Tarantola (2003), is:
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\[ \Phi(m, m_0) = r^T C_d^{-1} r + \alpha m^T L^T L m + \sum_k \lambda_k \| \psi_k (m, m_0) \|^2 \]  \hspace{1cm} (2)

where \( m_0 \) is the starting model, \( r = d - g(m) \) are data residuals, \( C_d \) is the data covariance matrix (supposing gaussian residuals), \( L \) can be any linear regularization operator and, the last term, is a sum of joint regularizers, each obtained with a particular link constraint to be satisfied by the two parameters to be inverted. An example for a suitable \( \psi_k \) is given by the Gardner’s Law, that relates P-velocity to density. An empirical law derived from well logs is also suited for building another \( \psi_k \) term. Finally, coefficients \( \alpha \) and \( \lambda_k \) are simple scalar weight terms: they can be increased or decreased to favor or reduce the influence of one particular term over \( \Phi \) optimization.

Geological constraints can influence the form of regularizers. Constraints from well logs can also enter into \( \Phi \) formulation.

SYNTHETIC TESTS

Several tests of the algorithm have been carried out on synthetic models and data. Processing of real data were also performed, but they are not shown here.

Synthetic Model and Data

For synthetic tests we used the 2-D model shown in Figure 2(a), which is resembling a salt dome buried into sediments. The synthetic gravity model and the synthetic resistivity model were derived using empirical relationships. The density model was derived using the Gardner’s law:

\[ \rho = k \sqrt{v} \]  \hspace{1cm} (3)

where \( \rho \) is density, \( v \) is P-velocity and \( k \) is a constant.

For the resistivity model we used an empiric log-linear law, verified with real well logs. The relationship takes the form:

\[ \ln(\rho) = a \cdot v + b \]  \hspace{1cm} (4)

where \( \rho \) is resistivity, \( v \) is P-velocity and \( a \) and \( b \) are parameters obtained from linear regression (in a log-linear space) of well log data (i.e. sonic velocity and resistivity).

For gravity and MT, the model parametrization followed an adaptive approach.

Especially for gravity and magnetotellurics, the effect of the earth parameter on measures decreases away from the measurement position. Therefore, whilst the velocity model was uniformly sampled along \( x \) and \( z \) directions, the grid spacing for density and resistivity models was set to increase away from the central region of interest (both along \( x \) and \( z \)). Furthermore, the density and resistivity models where extended laterally, relatively to the velocity one, to take into account border effects.

Synthetic seismic traces were generated using a finite-difference acoustic method and the velocity distribution of Figure 2(a) (FB data and post-migrated CIG data were both available). On the other side, using synthetic density and resistivity models, we obtained respectively gravity field data and apparent resistivity data (TE and TM mode, amplitude and phase).

Inversion of Synthetic Data

Joint inversion was performed iteratively, with final models from one iteration feeding the next iteration.

![Figure 2: Synthetic velocity model [m/s] (a) and corresponding PSDM stack (b).](image)

![Figure 3: Start velocity model [m/s] (a) and corresponding PSDM stack (b).](image)
As starting velocity model, a vertical gradient was chosen (see Figure 3(a)). This model was then converted to density or resistivity through the same empirical law used for synthetic model transformation.

After each JI iteration, we performed a Pre-Stack Depth Migration (PSDM) with the current estimated velocity model. Stack interpretation provided the layer shapes that could optionally be used to drive and/or differentiate the action of smoothing operators in the successive iteration. Layer shapes can also be useful for choosing to invert only some regions of the model instead of the whole model; they can also be used to invert CIG data in layer stripping mode.

The velocity model is changed during each inversion iteration and so do the layer shapes and positions. For this reason, if CIG data have to be used into the successive iteration, a new PSDM have to be performed, to pick the updated CIG curves.

![Image of final density model](image1)

Figure 4: Final density model [Kg/m³] (a), velocity model [m/s] (b) and final PSDM stack (c) after 5 iterations of Seismic-Gravity JI.

![Image of final resistivity model](image2)

Figure 5: Final resistivity model [Ω·m] (a), velocity model [m/s] (b) and final PSDM stack (c) after 7 iterations of Seismic-MT JI.
CONCLUSIONS

The Joint-Inversion velocity model building workflow showed various advantages over traditional approaches. Some of these are:

1. The simultaneous inversion of different geophysical data, improved quality of seismic migrations relative to the ones obtained with the seismic method only (see Figure 6(b));

2. Joint Inversion is able to achieve better model and stack quality in considerably fewer iterations. During our synthetic tests, we noticed that the total number of JI iterations were always no more than half the number of iterations we had to perform using the seismic method only. With seismic inversion alone, we had to perform 18 iterations whilst, with seismic-gravity JI we performed 5 iterations and, with seismic-MT JI, we carried out 7 iterations;

3. Joint Inversion is more stable in relation to a single domain inversion. Additional constraints given by link functions reduce the probability of artifact generation;

4. Joint Inversion can reduce the intrinsic non-uniqueness of inverse problems because the joint objective function is less subject to local minima (see Figure 7).

The proposed methodology could, theoretically, be extended also to the case of a Triple Joint Inversion involving seismic, gravity and MT data altogether.

REFERENCES


Dell’Aversana, P., S. Morandi, M. Buia, and D. Colombo, 2002, Pre-stack depth migration of global offset data integrated with high resolution magnetotelluric and gravity: Presented at the Annual International Meeting.

Kosloff, D. et al., 1996, Velocity and interface depth determination by tomography of depth migrated gathers: Geophysics, 61, 1511–1523.


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