Direct nonlinear traveltime inversion in layered VTI media
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Summary

We present a scheme for direct nonlinear inversion of picked moveout traveltimes in block-layered isotropic or VTI media. Using constant ray parameter traveltime differences between successive events allows inversion for velocity parameters of individual layers with no need for explicit layer stripping. We derive explicit closed-form inversion equations that allow this scheme to be implemented as a direct nonlinear method, rather than requiring iterative linearization. The method honors both anisotropy and ray bending effects more accurately than most conventional velocity analysis techniques.

Introduction

Conventional anisotropic velocity analysis methods for layered VTI media (Alkhalifah, 1997; Siliqi et al., 2003) usually scan midpoint-gather data over families of closed-form multiparameter moveout traveltime functions, trying to optimize some measure of similarity over offset such as semblance. The moveout curves used are usually parameterized in terms of effective velocity parameters that are in turn then inverted for interval velocity parameters using extended Dix equations derived from small-offset traveltime approximations. The accuracy of interval parameters derived this way is limited, particularly for wide-angle data, by the inability of the particular closed-form traveltime approximations to fit the details of anisotropic ray bending through many layers, and also by the inaccuracy of the small-offset approximations used for inversion from effective parameters to interval parameters.

One can pose the problem of inverting moveout traveltimes in a layered VTI medium as a form of tomography, and formulate solution methods based on iterative linearized updates of initial parameter estimates. Here we develop an alternative direct inversion method based on nonlinear equations relating traveltime differences across layers to underlying interval velocity parameters.

Method

Direct traveltime inversion requires that the medium be laterally invariant, so that the ray parameter \( p \) will be an invariant along a given ray. One can then identify points on successive events that have the same slope value \( p = dt / dx \). The traveltime and offset differences between two such points then depend only on the medium properties of the particular layer bounded by the two events. This is illustrated in Figure 1.

Claerbout (1978) proposed using this method for direct estimation of interval velocities in layered isotropic media. For that case one can use the simple inversion formula

\[
\frac{v^2}{p} = \frac{\Delta t}{\Delta t_0}
\]

where \( \Delta t \) is the offset difference, \( \Delta t_0 \) is the travelt ime difference, and \( v \) is the isotropic interval velocity.

Traveltimes in a layered VTI medium can be parameterized by the vertical traveltime \( t_0 \), the interval moveout velocity \( v_n \), and the interval horizontal velocity \( v_x \). To derive anisotropic constant-\( p \) inversion equations analogous to the isotropic inversion in equation (1), we begin with the simple relation between stepouts and traveltime for a ray segment in a single layer:

\[
\Delta t = p \Delta x + q \Delta z = p \Delta t_0 + q v_z \Delta t_0 \quad (2)
\]

Here \( p \) and \( q \) are the horizontal and vertical stepouts, or equivalently, components of the slowness vector, \( \Delta t_0 \) is the traveltime increment, \( \Delta x \) and \( \Delta z \) are the horizontal and vertical distance incremental, \( \Delta t_0 \) is the zero-offset traveltime increment, and \( v_z \) is the vertical velocity.

Following Douma and van der Baan (2006), this can be rewritten as

\[
\Delta t = v_z \Delta t_0 \left[ q - p \frac{\partial q}{\partial p} \right] \quad (3)
\]

\[
\Delta x = -v_z \Delta t_0 \frac{\partial q}{\partial p} \quad (4)
\]

To relate \( p \) and \( q \) we then use the approximate VTI P-wave dispersion relation given by Alkhalifah (1998):

\[
q = \frac{1}{v_z} \sqrt{1 - \frac{1 - v_z^2 p^2}{1 - (v_x^2 - v_z^2) p^2}}. \quad (5)
\]

Using equation (5) to eliminate \( q \) from equations (3) and (4) then gives (Douma and van der Baan, 2006)

\[
\Delta t = \frac{\Delta t_0}{ap^3} \left( \alpha^2 \beta^2 + \beta^2 - \alpha^2 \right) \quad (6)
\]

\[
\Delta x = \frac{\Delta t_0}{ap^3} \left( \beta^2 - \alpha^2 \right) \quad (7)
\]
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where for concision we define the factors

\[ \alpha^2 \equiv 1 - v_n^2 p^2 \quad (8) \]

\[ \beta^2 \equiv 1 - (v_x^2 - v_n^2) p^2 \quad (9) \]

We then solve equations (6) and (7) for the velocity parameters \( v_n \) and \( v_x \) to give the desired inversion equations

\[ v_n^2 = \frac{(\Delta t_0^2 - \Delta \tau^2)^2}{\Delta t_0^2 p^3 \Delta \tau} \quad (10) \]

\[ v_x^2 = \frac{\Delta t_0^2 (p \Delta x - \Delta \tau) + \Delta \tau^3}{\Delta t_0^2 p^3 \Delta x} \quad (11) \]

where again for concision in notation we define

\[ \Delta \tau \equiv \Delta t - p \Delta x. \quad (12) \]

To find the interval velocity parameters from the moveout traveltime curves one first needs to find \( p \) values along each curve by differentiating the traveltimes over offset. Then, for any given \( p \) value, one can determine the corresponding \( \Delta x \) and \( \Delta t \) values. This gives the needed information to use equations (10) and (11) to estimate the interval velocity parameters. This inversion scheme is usually highly overdetermined, because one can derive inverted parameter estimates for many different \( p \) values. Even with perfect traveltime data, the different estimates will not be identical because we used an approximate dispersion relation in deriving the inversion equations, but the errors are usually very small.

As an illustrative example, Figure 2 shows inversion estimates for a range of \( p \) values (plotted here as equivalent phase angles) derived from exact traveltimes computed for a single homogeneous VTI layer. The parameter estimates produced are accurate to within 0.1\% of the correct values over a wide range of angles. Note that very small angles are less accurate, because the inversion equations (10) and (11) become singular at vertical incidence.

For realistic cases, one usually picks residual traveltimes after an initial moveout or time migration with an approximate velocity model. We show a simple example in Figure 3 of a synthetic data set with ten layers, each 0.5 km thick. An initial isotropic ray-traced moveout correction was applied to the data. This left large residual moveout errors, as seen in the Figure 4a, which shows a window around the last four events after the initial isotropic moveout. The residual event traveltimes were then automatically picked and inverted. The anisotropic velocity parameters derived by inverting these picked traveltimes are shown in Figure 5. The parameter curves represent a trimmed mean of the inversion values over a range of \( p \) values. The inverted \( v_n \) and \( v_x \) values are accurate to within less than 3\% error everywhere. Figure 4b shows anisotropic ray-traced moveout correction using the inverted parameter functions applied to the same data window as in Figure 4a. The gathers are now nearly perfectly flattened.

Discussion and conclusions

Alternative implementations for this type of inversion are possible. The direct inversion equations (10) and (11) could also be used on slant-stacked \( (\tau - p) \) data. In that case, rather than differentiate traveltime over offset to find ray parameter \( p \), one differentiates over \( p \) to find offset \( x \). It may also be possible to extend this inversion approach to layered orthorhombic media (Ilya Tsvankin and Xiaoxiang Wang, pers. comm.).

The direct inversion method potentially handles both anisotropy and ray bending effects very accurately. The largest practical limitations in using this inversion scheme appear to be our ability to pick events accurately, and our ability to estimate accurate slopes from the picked traveltime curves. Application of this approach is also always limited by the underlying assumptions of a block layered medium with limited lateral velocity parameter variation, so it is of most use in the early stages of processing, and in areas of relatively undistorted sediments.

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Figure 1: Geometry of common ray segment between two events (after Claerbout, 1978).

Figure 2. Single layer inversion of exact traveltimes. (a) Estimated moveout velocity $v_m$ as a function of phase angle. Correct value is 4.3081 km/s. (b) Estimated horizontal velocity $v_x$ as a function of phase angle. Correct value is 4.7329 km/s. Note that both parameter estimates are accurate to within 0.1% over a wide range of angles.

Figure 3: Synthetic midpoint gather used for inversion example. Maximum offset is 8 km.
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Figure 4: Enlarged images of last four events in synthetic data. (a) After initial isotropic ray-traced moveout correction (b) After anisotropic ray-traced moveout correction using parameters derived from traveltime inversions.

Figure 5: Multilayer inversion of picked traveltimes. Estimated velocity functions are in red and exact model velocity functions are in black. Errors are everywhere less than 3%. (a) Interval moveout velocity $v_n$, (b) Interval horizontal velocity $v_x$. 
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