A three-dimensional multiplicative-regularized non-linear inversion algorithm for cross-well electromagnetic and controlled-source electromagnetic applications

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SUMMARY

We present three-dimensional inversion algorithms for inversion of cross-well electromagnetic and controlled-source electromagnetic (CSEM) data. The inversion is accomplished with a Gauss-Newton technique where the model parameters are forced to lie within their upper and lower bounds by means of a non-linear transformation procedure. The Jacobian is computed using an adjoint method. A line search method is employed to enforce a reduction of the cost function at each iteration. To improve the conditioning of the inversion problem, we use one of two different regularization schemes. The first is an $L_2$-norm regularization scheme, which provides for a smooth solution. The second is the weighted $L_2$-norm regularization scheme that can provide a sharp reconstructed image. The trade-off parameter that provides a relative weighting between the data and the model-constraint part of the cost function is determined automatically using the multiplicative regularized cost function to enhance the robustness of the method.

INTRODUCTION

Electromagnetic (EM) methods are one of the important tools for appraisal of a reservoir because of their sensitivity to conductivity which is a function of the fluid saturation. One of the commonly used EM techniques is single-well induction logging measurement. This technique is employed both as a wireline measurement and as a measurement while drilling to estimate near wellbore conductivity. This induction logging measurement has a sensitivity of up to a few meters from the well and is a function of the separation between the transmitter and receiver, the frequency of operation and the resistivity distribution.

To reach deeper into the reservoir, a cross-well EM technology was developed; see Wilt et al. (1995) and Spies and Habashy (1995). The system operates very similar to the single-well logging measurement, however, with transmitters (magnetic dipoles) and receivers (magnetic fields) deployed in separate wells and at a lower frequency of operation. After the data have been collected, an inversion process is applied to convert the EM signals to a conductivity distribution map of the region between the wells using a non-linear inversion algorithm.

Another type of an EM measurement that can reach deeper into the reservoir, is the so-called marine controlled-source electromagnetic (CSEM) method. The depth of investigation of this CSEM measurement is even larger than the cross-well measurement. This method recently received increased attention as a means for hydrocarbon exploration; see, for example, MacGregor and Sinha (2000), Eidesmo et al. (2002), and Johansen et al. (2005). The interest resulted from the technique’s ability to directly detect the presence of thin hydrocarbon-bearing layers. Initially, the data were analyzed by plotting the amplitude of the electric field versus source-receiver offsets, after normalizing the amplitude of the electric field that was acquired over a possible hydrocarbon prospect by the amplitude of the electric field measured over a similar non-hydrocarbon-bearing area (Eidesmo et al., 2002). Because the presence of hydrocarbon would increase the amplitude of the measured electric field, the normalized value will be greater than unity for areas containing resistive anomalies, and unity or less for non-hydrocarbon-bearing zones. Although this method will provide information on the presence of hydrocarbons, as well as some information on the horizontal location and extent of the reservoir, it is difficult to discern the reservoir’s depth or the true geometry. To provide this additional information, one will need to employ a full non-linear inversion approach.

We present a rigorous three-dimensional (3D) inversion algorithm. Unlike in Newman and Alumbaugh (1997), Abubakar and van den Berg (2000), Commer and Newman (2006), Mackie et al. (2007), Zhdanov et al. (2007) and Plessix and Sman (2007), where the minimization approaches are based on non-linear conjugate gradient (CG) or quasi-Newton techniques, we employed a Gauss-Newton minimization approach as described in Habashy and Abubakar (2004) using a multiplicative cost function (van den Berg et al., 1999). The Gauss-Newton approach is well-known to have higher convergence rates than non-linear CG or quasi-Newton methods. On the other hand, the Gauss-Newton method can be more expensive because it requires the computation and inversion of a Hessian matrix. By using the multiplicative cost function we do not need to determine the so-called regularization parameter in the optimization process; hence, the algorithm is fully automated. Further, the algorithm is equipped with two different regularization functions to produce either a smooth (using a standard $L_2$-norm function) or a blocky (using a weighted $L_2$-norm function) conductivity distribution; see van den Berg and Abubakar (2001). Moreover, in order to enhance the robustness of the algorithm, we incorporated a non-linear transformation for constraining the minimum and maximum values of the conductivity distribution. A line-search procedure for enforcing the error reduction in the cost function in the optimization process is also employed. We applied the inversion scheme to synthetic and field datasets both for cross-well and CSEM configurations.

THEORY

We consider a discrete non-linear inverse problem described by the following operator equation:

$$
\mathbf{d}^{\text{obs}} = \mathbf{s}(\mathbf{m}),
$$

where $\mathbf{d}^{\text{obs}} = [\mathbf{d}^{\text{obs}}(r_i^S, r_j^R, \omega_k), i = 1, 2, \cdots, I; j = 1, 2, \cdots, J; k = 1, 2, \cdots, K]^T$ is the vector of measured data and $r_i^S$, $r_j^R$, and
\( \omega \) are the source position vector, the receiver position vector and the frequency of operation. The superscript \( T \) denotes the transpose of a vector. The vector \( s(m) \) represents data computed by solving Maxwell equations:

\[
\nabla \times \left( \frac{1}{\sigma(r)} \nabla \times H(r) - i\omega \mu H(r) \right) = \nabla \times \left( \frac{1}{\sigma(r)} J(r)^S \right),
\]

where \( \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \) denotes the spatial differentiation operator, \( \sigma \) is the electrical conductivity, \( r \) is the spatial position, \( H \) is the magnetic field vector, \( i^2 = -1 \), \( \mu \) is a constant magnetic permeability, and \( J \) is the source vector. In the cross-well EM problem, the source is a magnetic dipole oriented parallel to the tow line and the receiver can be any component of the vector magnetic field that is also parallel to the borehole axis. In the CSEM problem the source is an electric dipole oriented parallel to the tow line and the receiver can be any component of the magnetic or electric vector field.

Equation (2) is discretized using the Lebedev grid employing a zero boundary condition at infinity. The computational domain is extended to infinity by using the optimal grid technique. The resulting linear system of equations (the stiffness matrix) is solved using a preconditioned quasi minimum residual (QMR) iterative technique. The preconditioning operator is constructed by using an integral equation operator for layered background medium. The details of the forward algorithm can be found in Zaslavsky et al. (2006).

To illustrate the accuracy of our forward modeling code we compare our 3D solution against a two-and one-half-dimensional (2.5D) solution (Abubakar et al., 2008) using a 2D model given in Figure 1. In this 2D configuration we have one resistive rectangular object of size 2 by 0.2 km and conductivity of 0.01 S/m embedded in a three-layer medium: air, water with conductivity 3 S/m, and an underburden beneath the seafloor with a conductivity of 1 S/m. The transmitter is located 60 m above the seafloor and receivers are located on the seafloor from \( x = -5 \) km to \( x = 5 \) km. The transmitter is an electric dipole oriented along the positive \( x \)-direction. Both in the 2.5D and 3D code the region of interest are discretized using uniform grids while the extension of the computational domains are done using non-uniform grids. In the 3D code we discretized the computational domain employing two grids: a 352 by 52 by 92 coarse grid and a 378 by 118 by 60 by 240 fine grid with a cell size of 17 m for the uniform grids. The grid used in the 2.5D code is a 378 by 118 coarse grid and a 698 by 288 fine grid. The comparisons of the inline fields (\( E_x \)) are shown in Figure 2. On the left panel of Figure 2, the amplitude and phase of the 3D solutions are shown in blue, while the 2.5D solutions are shown in red. In the right plots of Figure 2, the differences between these two solutions are shown as a blue solid line for the coarse grid and a dashed red line for the fine grid. We observed that, except for points that are close to the transmitter position, we only observed a small difference (up to 5% in amplitude and 1.7 degrees in phase for the coarse grid and up to 0.8% in amplitude and 0.9 degrees in phase for the fine grid). The CPU time for the 3D solution is 594 s using the coarse grid and 1756 s using the fine grid on a PC with a 3.04 GHz processor.

The unknown vector model parameters in (1) are defined as follows:

\[
m = [m(x_l, y_l, z_q), \quad l = 1, \ldots, L; \quad t = 1, \ldots, T; \quad q = 1, \ldots, Q],
\]

where \( x_l, y_l \) and \( z_q \) denote the center of the discretization cell. We assume that there are \( I \times J \times K \) number of data points in the experiment and that the configuration can be described by \( L \times T \times Q \) model parameters. The unknown model parameter \( m(r) = \sigma(r)/\sigma_0(r) \) is the normalized conductivity where \( \sigma_0(r) \) is the conductivity distribution of the initial model used in the inversion.

We pose the inverse problem as the minimization problem of a multiplicative cost function (van den Berg et al., 1999). Hence, at the \( n \)th iteration, we reconstruct \( m_n \) that minimizes

\[
\Phi_m(m_n) = \phi^d(m_n) \times \phi_m^m(m),
\]

where \( \phi^d \) is the data misfit, given by:

\[
\phi^d(m_n) = \frac{1}{2} \sum_{k=1}^{K} \sum_{i=1}^{I} \sum_{j=1}^{J} \left[ W_{d,i,j,k} \left( d_{i,j,k}^{\text{obs}} - s_{i,j,k}(m_n) \right) \right]^2,
\]

in which \(| \cdot | \) denotes the absolute value and \( W_{d,i,j,k} \) is the data weighting matrix.

The non-zero regularization function \( \phi_m^m \) is a measure of the variation of the model parameters in the inversion domain and
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The cost function is given by,

$$
\phi_n^m(m) = \sum_{k=x,y,z} \alpha_k \int_D b^2_k(x,y,z) \left\{ |\partial k m(x,y,z)|^2 + \delta_{k,n} \right\} \, \mathrm{d}x \mathrm{d}y \mathrm{d}z,
$$

where $\alpha_k$ for $k = x, y, z$ are the smoothing ratio factors and the weights $b_{k,n}(x,y,z)$ are given by

$$
b^2_{k,n}(x,y,z) = \frac{1}{V} \int_D \left\{ |\partial k m_n(x,y,z)|^2 + \delta_{k,n} \right\} \, \mathrm{d}x \mathrm{d}y \mathrm{d}z
$$

for the $L_2$-norm regularizer and

$$
b^2_{k,n}(x,y,z) = \frac{1}{V} \frac{1}{|\partial k m_n(x,y,z)|^2 + \delta_{k,n}} , \quad V = \int_D \mathrm{d}x \mathrm{d}y \mathrm{d}z
$$

for the weighted $L_2$-norm regularizer as introduced in van den Berg and Abubakar (2001). The $L_2$-norm regularizer is known to favor smooth profiles, while the weighted $L_2$-norm regularizer is known for its ability to preserve edges. Note that for the $L_2$-norm regularizer, the weight $b_{k,n}(x,y,z)$ is independent of the spatial position. The positive parameter $\delta_{k,n}$ is a constant that is chosen to be equal to:

$$
\delta_{k,n} = \frac{\phi^d(m_n)}{(\Delta k)^2}, \quad k = x, y, z,
$$

where $\Delta x$, $\Delta y$ and $\Delta z$ are the widths of the discretization cell in the $x$, $y$, and $z$ directions. Note that the presence of $\delta_{k,n}$ ensures that the regularization factor will be non-zero. This weighted $L_2$-norm regularization factor belongs to the same class as the well-known total variation regularization (Dobson and Santos, 1996; Vogel and Oman, 1996; and Farquharson and Oldenburg, 1998) and the so-called focusing regularization function (Portniaguine and Zhdanov, 1999). Although this weighted $L_2$-norm regularization cost function has all the advantages of the total variation regularization function, it is still a quadratic function. It means that it has a numerically defined gradient; hence, it is more suitable to be used with a Gauss-Newton approach.

To solve (4) we employ a Gauss-Newton minimization approach. At the $n$th iteration, we obtain a set of linear equations for the search vector $p_n$ that identifies the minimum of the approximated quadratic cost function, namely,

$$
\mathcal{H}_n \cdot p_n = -g_n,
$$

where the Hessian matrix is given by

$$
\mathcal{H}_n = \phi_n^m(m_0) \left\{ \mathcal{J}_n^T \cdot \mathcal{W}_d^T \cdot \mathcal{W}_d \cdot \mathcal{J}_n + \Delta_n \right\}
$$

$$
+ \phi^d(m_0) \mathcal{Q}^{(1)}(m_0)
$$

$$
+ \left[ \mathcal{Q}^{(1)}(m_0) \cdot m_0 \right] \left\{ \mathcal{J}_n^T \cdot \mathcal{W}_d^T \cdot \mathcal{W}_d \cdot \left[ \mathcal{d}^{\text{obs}} - s(m_0) \right] \right\}
$$

$$
+ \left\{ \mathcal{J}_n^T \cdot \mathcal{W}_d^T \cdot \mathcal{W}_d \cdot \left[ \mathcal{d}^{\text{obs}} - s(m_0) \right] \right\} \left[ \mathcal{Q}^{(1)}(m_0) \cdot m_0 \right].
$$

To make the Hessian matrix non-negative definite, in (11), we neglect the second-order derivative of the cost function, given by the matrix term $\Delta_n$, as well as the the third and the fourth terms. Further, we also used the fact that $\phi^m(m_n) = 1$. By doing so, the Hessian matrix becomes

$$
\mathcal{H}_n \approx \mathcal{J}_n^T \cdot \mathcal{W}_d^T \cdot \mathcal{W}_d \cdot \mathcal{J}_n + \phi^d(m_0) \mathcal{Q}^{(1)}(m_0).
$$

The matrix $\mathcal{J}_n$ is the Jacobian matrix and is given by

$$
\mathcal{J}_{i,j,k,s,q,n} = \frac{\partial S_{i,j,k,s,q,n}}{\partial \sigma_{i,j,k,s,q,n}}
$$

$$
\sigma_{i,j,k,s,q} = \left. \frac{\partial S_{i,j,k,s,q,n}}{\partial \sigma_{i,j,k,s,q,n}} \right|_{m=m_0}
$$

where $\partial S/\partial \sigma$ is obtained using the adjoint approach. The first and the second derivative of the regularization function in (6) with respect to $m$ evaluated at $m = m_0$ are given by

$$
\left[ \mathcal{Q}^{(1)}(m_0) \right]_{i,j,q} = \sum_{k=x,y,z} \alpha_k \left\{ \partial k b_{k,n} \left[ \partial k v^i \right]_{i,j,q} \right\}_{k=x,y,z}.
$$

The multiplicative cost function given in (4) is equivalent to the following standard cost function:

$$
\Phi_n(m) = \phi^d(m) + \lambda_n \phi^m(m),
$$

with the choice of the regularization parameter $\lambda_n$ equal to

$$
\lambda_n = \frac{\phi^d(m_0)}{\phi^m(m_0)} = \phi^d(m_0),
$$

because $\phi^m(m_0)$ is equal to unity.

This multiplicative cost function will minimize the regularization factor with a large weight at the beginning of the optimization process because, the value of $\phi^d(m)$ is still large. In this case, the search direction is predominantly a steepest descent, which is more appropriate to use in the initial steps of the iteration process since it has the tendency of suppressing large swings in the search direction. As the iteration proceeds, the optimization process will gradually reduce the error in the data misfit while the regularization factor $\phi^m(m)$ remains at a nearly constant value close to unity. In this case, the search direction corresponds to a Newton search direction, which is more appropriate to use as we get closer to the minimum of the data misfit cost function $\phi^d(m)$ where the quadratic model of the cost function becomes more accurate. If noise is present in the data, the data misfit cost function $\phi^d(m)$ will plateau to a certain value determined by the signal-to-noise ratio; hence, the weight, $\lambda_n$, on the regularization factor will be non-zero. In this way, the noise will, at all times, be suppressed in the inversion process and the need for a larger regularization when the data contains noise will be automatically fulfilled as suggested by Rudin et al. (1992) and Chan and Wong (1998).

After the Gauss-Newton search vector $p_n$ is obtained by solving the linear system of equations in (10) using a conjugate gradient least square (CGLS) technique, the unknown model parameter is updated using a non-linear transformation and the line-search procedure. The details of the non-linear transformation procedure and the line search procedure can be found in Habashy and Abubakar (2004).
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Figure 3: Horizontal slices (xy-plane) for \( z = 562.5, \cdots, 637.5 \) m of the conductivity distribution of the true model.

Figure 4: Vertical slices (xz-plane) for \( y = -37.5, \cdots, 37.5 \) m of the conductivity distribution of the true model.

Figure 5: Horizontal slices (xy-plane) of the inversion results after seven Gauss-Newton iterations.

Figure 6: Vertical slices (xz-plane) of the inversion results after seven Gauss-Newton iterations.

NUMERICAL EXAMPLES

As a numerical example we consider a water-flood experiment in the cross-well configuration. The synthetic model is shown as horizontal (xy plane) and vertical (xz plane) slices in Figures 3 and 4. In this model we have three layers with resistivity of 5, 8, and 1 ohm-m. The thickness of the middle layer is 30 m. The survey is done using two wells. The transmitter well is located at \( x = 0 \) and \( y = 0 \) while the receiver well is located at \( x = 200 \) m and \( y = 0 \). The water injection is carried out in the receiver well. The resistivity of the waterflooded region is 0.5 ohm-m.

We use 41 transmitter and 41 receiver stations located from \( z = 500 \) to \( z = 700 \). The transmitters are magnetic dipoles oriented in the positive \( z \)-direction operating at 125 Hz while the receivers measure the \( H_z \) component only. The synthetic data are corrupted with 2% random white noise.

We limited our inversion domain from \( x = -50 \) m to \( x = 250 \) m, \( y = -50 \) m to \( y = 50 \) and \( z = 500 \) m to \( x = 700 \). The total number of inverted parameters is equal to 60 by 20 by 40. The grid used in the forward code is 118 by 74 by 98.

As the initial model, we employed the three layered model, i.e., the true model without the waterflooded region. The smoothing ratio factors \( \alpha_x, \alpha_y \), and \( \alpha_z \) are chosen to be equal to unity.

The inversion results after seven Gauss-Newton iterations are given in Figures 5 and 6. The data misfit has been reduced from 12.10% to 2.55% (which is about the noise level). As these figures show, we obtain a reasonable inverted model despite the limited dataset that was used in the inversion.

In the presentation, we plan to show inversion results using more than two cross-well surveys as well as inversion results for the CSEM configurations.
EDITED REFERENCES
Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2008 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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