Seismic properties of unconsolidated sands: Tangential stiffness, Vp/Vs ratios and diagenesis
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Summary

Ultrasonic and well log measurements of unconsolidated sands indicate that observed velocities are lower than predicted from Hertz-Mindlin contact theory for random packing of identical grain sizes. In particular, the Vp/Vs ratios are significantly higher than predicted from models. This discrepancy is important to understand prior to AVO analysis of unconsolidated sands. We perform a sensitivity study of the contact model and find that the model most likely to fit observed Vp, Vs and Vp/Vs is the model where the effective elastic constants are calculated by assuming some of the grains to have no tangential stiffness. In general, contact theories relate effective bulk and shear moduli to tangential and normal stiffness, and the choice of tangential contact stiffness model changes the effective moduli of the grain pack. Bachrach et al. (2000) showed that Vp/Vs ratio in unconsolidated sands is related to the percent of grains that have no tangential contact stiffness, while grain angularity (i.e., accounting for non-uniform contact radius) may reduce both the bulk and shear moduli. In this paper we analyze the elasticity and AVO response of specific deep water reservoir sands and find that their Poisson’s ratio is consistent with that of sands at low effective stress. This may be supported by laboratory measurements of stress distribution in granular media. We also note that at higher pressures, the effect of angularity is of second order compared to the effect of zero tangential contact stiffness.

Introduction

It has been found that Hertz-Mindlin contact theory overpredicts velocities and underpredicts Poisson’s ratios (Vp/Vs) in unconsolidated sands (e.g., Bachrach et al., 2000, Zimmer et al., 2002). Dvorkin and Nur (1996) found that most effective medium models overestimate Vs for poorly consolidated sands. It has been suggested that this might be due to neglecting slip and/or rotational degrees of freedom of sand grains under shear loading. Grain angularity has also been found to cause lower velocities than predicted by Hertz-Mindlin contact theory (Bachrach et al., 2000). Avseth et al. (2005) have assumed this to be a second order effect for poorly to moderately consolidated sediments in situ at burial depths of several hundred meters to a few kilometers. However, in this paper we show that unconsolidated sands of Miocene age, located at burial depths of about 700 m, have higher measured Poisson’s ratios than predicted by Hertz-Mindlin contact theory. According to this theory, the effective dry frame Poisson’s ratio is only a function of the mineral Poisson’s ratio.

Spencer et al. (1994) conducted laboratory measurements and found that the effective Poisson’s ratio in unconsolidated sands, contrary to the theory, does not depend on mineralogy. They suggested the discrepancy to be caused by the physical state of the grain assemblage. However, they were unable to identify the specific factors controlling the Poisson’s ratios of unconsolidated sands. Bachrach et al. (2000) found that the main reason for higher effective Poisson’s ratios than mineral Poisson’s ratios was slip at the grain contacts. In this paper we show how this effect must be taken into account before doing model based AVO analysis in unconsolidated sands.

Theory

Walton (1987) showed that for a dry, dense, random pack of identical elastic spheres, the effective bulk (K_eff) and shear (G_eff) moduli are:

$$K_{eff} = \frac{n(1-\phi)}{12\pi R_g} S_n, \quad G_{eff} = \frac{n(1-\phi)}{20\pi R_g} (S_n + 1.5S_I) \quad (1)$$

where S_n and S_I are the normal and shear stiffness of a two-grain contact, respectively; n is the average number of contacts per grain (coordination number); \( \phi \) is porosity; and \( R_g \) is the grain radius.

Note that equation (1) state that the effective bulk modulus is linearly related to the normal stiffness and the coordination number, whereas the effective shear modulus is also linearly related to the tangential stiffness. This is true for any contact model. In general the contact stiffness is related to the loading path and boundary conditions at the grain contact. One particular contact model is the Hertz-Mindlin model which assumes that grain contacts are first exposed to normal loading, while tangential forces are applied afterwards. Then the contact stiffnesses are given by (Mavko et al., 1998):

$$S_n = \frac{4aG}{1-\sigma}, \quad S_I = \frac{8aG}{2-\sigma}, \quad (2)$$

where a is the radius of the contact area between two grains and G and \( \sigma \) are the grain shear and Poisson’s ratio, respectively. The radius a is related to the confining force F and the contact radius of curvature \( R_c \) by the following:
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The confining force $F$ can be expressed through hydrostatic effective pressure $P$ (e.g., Marion, 1990):

$$F = \frac{4\pi R^2 P}{n(1-\phi)}.$$  \hspace{1cm} (4)

Bachrach et al. (2000) showed that the effective Poisson’s ratio can be expressed in terms of the tangential and normal stiffness as follows:

$$\sigma_{eff} = \frac{S_n - S_t}{4S_n + S_t}. \hspace{1cm} (5)$$

When the tangential stiffness between the grains is negligible ($S_t = 0$; e.g., in loose sands with water-lubricated grain contacts), the equation above implies that the effective Poisson’s ratio is 0.25 or the effective $V_p/V_s$ is 1.73 (Walton, 1987). The effective Poisson’s ratio observed in sands at low effective stress was 0.15 ($V_p/V_s = 1.56$) which was modeled by Bachrach et al. (2000) as an aggregate where some of the grains have zero tangential stiffness while other had tangential stiffness predicted by Hertz-Mindlin. This value was also suggested by Winkler (1983). Spencer et al. (1994) found that effective Poisson’s ratios in unconsolidated sands and sandstones vary between 0.091 and 0.237.

Rock Physics Analysis

Hertz-Mindlin theory can be applied to predict velocity versus effective pressure. Figure 1 shows P-wave and S-wave velocities as well as $V_p/V_s$ ratios as a function of pressure, for both dry and saturated data. For the dry data we have included data from Zimmer et al. (2002). The saturated data comprise both data from Zimmer et al. and average values for clean turbidite sand intervals of Miocene age. First of all, note that all data fall closer to the line where we assume zero tangential stiffness ($S_t = 0$ in equation 1). Second, there is a nice match between the well log data from the deep-water reservoir sands and the laboratory data of Zimmer et al. For the dry data, it is interesting to note that the $V_p/V_s$ ratio is independent of pressure. In our case we are dealing with quartz, still the data plot far above 1.5 (dashed line), which is the expected value for quartz sands. In stead, the $V_p/V_s$ ratio is closer to 1.75 due to the reduced shear stiffness. For brine saturated sands, the $V_p/V_s$ ratios range from above 3.5 to about 2.5. We have done similar studies for Oligocene sands, but these fall closer to the contact theory, with $V_p/V_s$ ratios of brine sands close to 2.0. We have also looked at the sensitivity of grain angularity, but found this to have a second order effect on $V_p/V_s$ ratios compared to the effect of reduced tangential shear stiffness.

In Figure 2 we include both Miocene sands and Oligocene sands superimposed on rock physics depth trends, where we use both original contact theory as well as modified contact theory with zero tangential shear stiffness. Even though the porosity values varies significantly, the Miocene data fall between the original contact theory and the modified with reduced shear stiffness, yet closer to the latter. The Oligocene data actually fall slightly above the velocities predicted by the original contact theory, but the $V_p/V_s$ ratios still show a nice match. The Oligocene sands may have some initial contact cement. This has been confirmed in the same target zone (Avseth et al., 2005). It is important to obtain correct rock physics depth trends during seismic exploration in immature areas with no or few well calibration points (Avseth et al., 2003).
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Figure 2: Rock physics depth trends compared to well log data from deep-water reservoir sands (only clean sand intervals included). Dashed lines represent original Hertz-Mindlin. Solid lines represent zero tangential stiffness. Note how Miocene sands (600-800m burial depth) fall closer to modified trends. The colorscale to the right is porosity. The depth zone 0-0.6km includes no data, but modeled overburden estimates.

Figure 3 shows rock physics templates (RPTs) for specific Miocene and Oligocene deep-water targets. For a detailed explanation of RPT technology, see Ødegaard and Avseth (2004) or Avseth et al. (2005). We have superimposed data from Zimmer et al. (2002) at 5 and 10 Mpa (upper plot), as well as data from Miocene deep-water reservoirs. There is a significant discrepancy between the data and the RPT models. This is because the RPTs do not take into account reduced tangential shear stiffness. In the lower RPT we include data from Zimmer et al. at 15 Mpa, and well log data from Oligocene sands. In this case we observe that the well log data better match the modeled brine sand trend at expected porosity values. The data from Zimmer et al., however, still fall off the template. This could indicate that initial consolidation of sands, causes zero slip boundary condition at the edges of the grain contact and thus tangential shear stiffness values closer to what we predict from Hertz-Mindlin contact theory. The sand samples of Zimmer et al, however, have not been exposed to any diagenetic consolidation.

It is clear that the effect of reduced tangential shear stiffness, as we have manifested in the Miocene sands above, will have an important impact on the seismic signatures. In Figure 4 we have done AVO modeling of a half-space model, with a reservoir sand capped by a shale. The rock physics properties of shales represent average Miocene shale values: Vp1=2100 m/s, Vs1=900 m/s and Rho1=2200 kg/m3. For the reservoir sand, we have made 4 scenarios: 1) Sand according to original contact theory saturated with brine; 2) Sand with zero tangential shear stiffness with brine; 3) and 4) are ditto with oil (20 API). It is remarkable how big the effect of reduced shear stiffness is. First of all, we note that the gradient becomes flat or even turn sign due to the reduced shear stiffness. This is because the shear stiffness approaches that of the shale, and we know that the AVO gradient is mainly controlled by the contrast in Vp/Vs ratio. The intercept, however, becomes more negative, with a higher amplitude. This is a result of the impedance drop due to the effect of reduced shear stiffness on the P-wave velocity. Figure 4 shows how important it is to take into account the correct rock physics model in AVO feasibility studies.
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Figure 4: AVO curves for brine (blue) and oil (red), both with original contact theory (dashed) and reduced tangential shear model (solid lines). Note the decrease in gradient and negative increase in intercept when we reduce shear stiffness. For the brine sands we observe a change from class II to class IV AVO anomaly. For the oil sands we go from class III to almost a class IV.

Discussion

The above rock physics analysis show that sand diagenesis and elasticity are intimately related, and follow the following patterns: for loose sands where no cementation is observed measured Poisson’s and Vp/Vs ratios can be modeled as an aggregate where some of the tangential stiffness at grain contacts is zero (or in other words, allowing slip at the grain boundaries), whereas when small amount of cement is introduced to the system it is evident that the Hertz-Mindlin model which assumes no slip at the boundaries is modeling better the effective aggregate. The observation in loose sands are consistent both in laboratory and field experiments at different scales and methods and expand different frequency ranges (surface seismic, log data and laboratory. Observation in photo-elastic granular media show that in such medium the stress is not uniformly distributed in the aggregate, rather it is localized in “stress chains” (Geng et al, 2001). This may support the model we have been using as when stress is localized, not all grain contacts have the same tangential stiffness and loading history.

Conclusions

We have shown that:
1. The elastic properties of unconsolidated sands are nicely matched by contact models where some of the grains have no tangential stiffness.
2. Slightly cemented sands are consistent with Hertz-Mindlin theory which assumes no slip at the grain boundaries
3. These different models, which yield different Vp/Vs ratios, should be taken into account when modeling AVO responses at different ages.

References

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EDITED REFERENCES

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