A New Pressure-Rate Deconvolution Technique Based on Pressure Derivatives for Pressure Transient Test Interpretation

Mustafa Onur, Istanbul Technical University, and Fikri J. Kuchuk, Schlumberger

Copyright 2010, Society of Petroleum Engineers

Abstract

In this paper, we present a new deconvolution method that removes the dependency of the deconvolved constant-rate drawdown responses to the initial reservoir pressure. As is well known, particularly the late-time portions of the deconvolved responses from the recent pressure-rate (p-r) deconvolution algorithms are dependent on the initial reservoir pressure. A small error in the initial reservoir pressure could make a significant difference in the late-time portions of the deconvolved responses that can easily lead to an incorrect interpretation model, particularly misinterpretation of the boundaries. The new method presented is based on pressure-derivative data rather than pressure data that are used in all published deconvolution algorithms. Using pressure-derivative data in deconvolution leads to a nonlinear least-squares objective function that is different from those used in the earlier deconvolution methods and eliminates the dependency of the deconvolved responses to the initial reservoir pressure. Hence, the new method minimizes incorrect interpretation due to an error or uncertainty in the initial reservoir pressure.

We apply the new method to both simulated and field pressure-transient data sets. The results show that the new method offers a significant advantage over the earlier deconvolution methods for pressure transient test interpretation in cases where the initial reservoir pressure is unknown or uncertain.

Introduction

During the last decade, pressure-rate (p-r) deconvolution algorithms have been improved significantly, and a few algorithms are now available via commercial well-test interpretation software. Pressure-rate deconvolution makes available an equivalent constant-rate drawdown pressure and its Bourdet derivative (derivative with respect to the natural logarithm of time, see Bourdet et al. 1989) to be used for system identification and interpretation. Recent publications demonstrate the uses of pressure-rate deconvolution for pressure transient wireline formation and well-test data as well as permanent pressure gauge data (von Schroeter et al. 2004, Levitan 2005, Ilk et al. 2006, Onur et al. 2008, Pimonov et al. 2009, and Kuchuk et al. 2010).

In all p-r deconvolution algorithms mentioned above, the value of initial reservoir pressure has to be known. If it is unknown or uncertain, the initial reservoir pressure must be treated as an unknown parameter to be determined by history matching measured pressure data. Although the earlier deconvolution algorithms, except the one described by Ilk et al., allow one to treat the initial reservoir pressure as an unknown parameter in deconvolution history matching, numerical experiments with these algorithms show that one could determine the appropriate value of the initial reservoir pressure only for some limited cases of pressure/rate data sets, e.g., consistent pressure/rate data sets with the deconvolution model assumed by the algorithms (for example see, Levitan et al. 2006 and Onur et al. 2008). The algorithms assume that the convolution integral given below is linear and the relationship between the flow rate \( q_m(t) \) and the wellbore pressure \( p_m(t) \) is given by Duhamel's convolution integral (van Everdingen and Hurst 1949):

\[
p_m(t) = p_0 - \int_0^t q_m(t - \tau) g(\tau) d\tau, \quad t \in [0, T],
\]

(1)

where \( p_0 \) is the initial reservoir, and \( T \) is the total duration of pressure/rate measurements. Equation 1 applies to test data sets from a single source/sink well in the reservoir, i.e., no interference for other wells. Hence, for such cases, \( q_m \) represents the measured flow rate at the source/sink well, whereas \( p_m \) represents the measured pressure either at the wellbore of the source/sink well or at the wellbore of an observation well. In Eq. 1, for a given time period, each wellbore/reservoir system has a characteristic unit-impulse function (or response) \( g(t) \) at the source/sink well or at the observation well where \( p_m \) is measured. The unit-impulse function \( g(t) \) is a time derivative of constant-unit-rate drawdown pressure change \( p_d(t) \), i.e., \( g(t) = dp_d(t)/dt \).
Equation 1 is usually referred to as pressure-rate convolution equation for a single source/sink system. In this work, we do not consider the multi-well convolution/deconvolution problem, although the new deconvolution method can readily be extended to the multi-well deconvolution considered previously by Levitan (2007) and von Schroeter and Gringarten (2009). It is also worth noting that in general, measurements are subject to some errors (measurement noise). There are many different sources of noise that can be encountered in wellbore pressure and flow rate measurements. Therefore, $p_m$ and $q_m$ in Eq. 1 normally contain measurement errors. For deconvolution, Eq. 1 is the measured wellbore pressure model and hence the wellbore pressure to be computed from Eq. 1 is to be denoted by $p_d(t)$.

As shown by Levitan et al. (2006), when inconsistent measured pressure/rate data sets (i.e., the data sets for which the linearity of the convolution integral given by Eq. 1 is violated), which may be normally encountered in practice, e.g., changing wellbore storage coefficient, varying skin, multi-phase flow in the reservoir, etc., are used in Eq. 1, then the deconvolved unit-rate drawdown responses obtained from Eq. 1 will be incorrect. Therefore, it seems that most of the time, we should apply the $p-r$ deconvolution based on Eq. 1 to the portions of pressure/rate data that are consistent with Eq. 1, e.g., the buildup portions and good quality drawdown portions of the same test sequence. Also as shown by Levitan et al. (2006), the pressure data from a single pressure buildup period do not contain enough information to estimate the initial reservoir pressure and the unit-rate wellbore storage coefficient, varying skin, multi-phase flow in the reservoir, etc., are used in Eq. 1, then the deconvolved unit-rate drawdown response function $p_d(t)$ and its Bourdet derivative at the same time. In other words, it is generally not possible to simultaneously determine the value of initial pressure and the response function from a single buildup period. As stated above, a small error in the initial reservoir pressure can make a significant difference on the late-time portion of the deconvolved response and yields an erroneous deconvolved response that can easily lead to an incorrect identification of the model, particularly misinterpretation of the boundaries. As shown by Levitan et al., it may be possible to determine the appropriate value of the initial reservoir pressure by a trial-and-error procedure which yields a consistent deconvolved response at late times from several buildup periods of the same test sequence. Perhaps, the most important limitation of this procedure is its requirement that the test sequence includes at least two consistent buildup periods.

In this work, in contrast to the earlier pressure-rate ($p-r$) deconvolution algorithms of von Schroeter et al. (2004), Levitan (2005), Ilk et al. (2006), and Pimonov et al. (2009), we propose a new deconvolution method based on an appropriate objective function using pressure-derivative data instead of pressure data to estimate (reconstruct) the unit-rate drawdown responses. Thus, the objective function proposed in this work for deconvolution history-matching does not depend on the initial reservoir pressure.

The paper is organized as follows: First, we present basic mathematical formulations used throughout this work and then the formulation of an objective function based on the use of Bourdet pressure derivative data (referred to as pressure-rate derivative deconvolution) to reconstruct the unit-rate drawdown responses. Finally, we present applications of the pressure-derivative-rate deconvolution method for one synthetic and two field pressure transient test examples.

**Mathematical Formulations: Pressure-Derivative-Rate Convolution**

Here and throughout, we let $p'_u(t)$ denote the Bourdet derivative of the constant-unit-rate drawdown pressure change $p_u(t)$ given as (Bourdet et al, 1989)

$$p'_u(t) = \frac{dp_u(t)}{d\ln t} = \int dp_u(t) = t g(t)$$

and similarly let $p'_m(t)$ denote the Bourdet derivative of measured pressure data, given as

$$p'_m(t) = \frac{dp_m(t)}{d\ln t} = \int dp_m(t) = t.$$  

The second equalities of Eqs. 2 and 3 follow directly from the chain rule.

In this work, our main objective is to reconstruct deconvolved constant-rate drawdown responses [$p_u(t)$ and $p'_u(t)$] using the Bourdet derivative or pressure-derivative (both will be used interchangeably in this paper) of the measured pressure data so that the requirement for a known initial reservoir pressure $p_0$ is eliminated. Two different approaches can be used to estimate deconvolved constant-rate responses. One approach is to use a convolution equation expressed only in terms of pressure-derivative data instead of pressure data. We refer to such a convolution equation as the pressure-derivative-rate convolution equation, and it can be derived starting from Eq. 1 by using Eqs. 2 and 3 as

$$p'_m(t) = -\frac{i}{0} q_m(t-\tau) \frac{dp'_u(\tau)}{d\tau} d\tau.$$  

Assuming $q_m(0) = 0$ and $p'_u(0) = 0$, it can also be written as

$$p'_m(t) = -\frac{i}{0} q_m(\tau) \frac{dp'_u(t-\tau)}{dt} d\tau.$$  

It is clear that the pressure-derivative-rate convolution equations given by Eqs. 4 and 5 are independent of the initial pressure, \( p_0 \). Eqs. 4 and 5 will be called the measured wellbore pressure-derivative models and hence, the wellbore pressure-derivative to be computed from Eqs. 4 and 5 will be called the computed wellbore pressure-derivative and is denoted by \( p'_c(t) \). As in pressure deconvolution, we can use the von Schroeter et al. transformation

\[
z_d(\xi) = \ln[\tau g(\tau)] = \ln[p'_m(\tau)]
\]

for given values of \( p_0, q_m, \) and \( z_d \) response function, to generate the computed pressure data \( p_c(t) \). Then, the computed pressure-derivative data \( p'_c(t) \) will be obtained by using the same numerical differentiation procedure that is also used to differentiate measured pressure data. The numerical differentiation procedure is given below, where we will drop the subscripts \( c \) and \( m \) from pressure and pressure-derivative because the same differentiation procedure will be used for both computed and measured data sets.

Equation 9 gives an approximation for pressure-derivative data at interior points. Although at the first data point of time \( t_1 \) and the last data point of time \( t_{N_p} \), one-sided difference approximations could be applied, however, we use pressure-derivative data computed only at interior time points using Eq. 9, thus there will be \( N_p - 2 \) derivative data points for nonlinear regression matching (i.e., \( N_d = N_p - 2 \)) which will be discussed in the next section.

Before closing this section, we make the following remarks: We can use Eq. 8 with the unit-slope assumption at the first node as applied by von Schroeter et al. (2004) or use a different (actually an approximate) formulation of the convolution equation given as


\[ p_c(t) = p_0 - q_m(t)p_a(\tau_1) - \int_{-\infty}^{\ln\beta_1} q_m(t - \xi) e^{\xi} d\xi \]

(13)

to eliminate the wellbore storage unit-slope assumption before the first node \([\varsigma_1 = \ln(\tau_1)]\) as applied by Levitan (2005). We have implemented both options. In addition, like von Schroeter et al., Levitan, and Pimonov et al., we assume that the flow rate \(q_m\) in Eq. 8 can be represented as stepwise constant: \(q_m(j) = \beta_j\), \(j = 1, 2, \ldots, N_q\), is the rate over a time interval \(\epsilon_j \leq t \leq \beta_j\). Like these authors, we also reconstruct the \(z_d\) function by choosing nodes \(\varsigma_k\) such that

\[-\infty = \varsigma_0 < \varsigma_1 < \varsigma_2 < \cdots < \varsigma_N = \ln(\beta_k),\]

(14)

and interpolate \(z_d\) linearly between values \((z_d(k) = z_d(\varsigma_k))\). In our applications, we use logarithmically equally spaced nodes for computation of the unknown function \(z_d(\varsigma)\), where the nodes \(\varsigma_k\) are generated from

\[\varsigma_k = \varsigma_1 + \frac{k-1}{N-1}(\varsigma_N - \varsigma_1); \text{ for } k = 1, \ldots, N.\]

(15)

A typical value of \(N\) which we use is 70 which works well for the deconvolution method proposed in this work. So, unless otherwise stated, \(N = 70\). The last node \(\varsigma_N\) is defined as \(\varsigma_N = \ln(T)\), where \(T\) is the elapsed time from the start of rate history through the last point of the pressure data selected for deconvolution. The first node \(\varsigma_1\) is usually chosen as less than or equal to the minimum elapsed time of the first pressure measurement. The method assumes \(\varsigma_1\) as \(t_{\min} = 10^{-3}\) hr by default. However, if \(t_{\min} > 10^{-3}\) hr is greater than the time \(t_1\) of the first pressure point, then \(t_{\min}\) is automatically changed to \(t_1\), i.e., we set \(t_{\min} = t_1\), and hence compute \(\varsigma_1 = \ln(t_1)\) to be used in Eq. 15, where we used \(t_{1_k} = 10^{-3}\) hr by default. However, if \(t_{\min} = 10^{-3}\) hr is greater than the time \(t_1\) of the first pressure point, then \(t_{\min}\) is automatically changed to \(t_1\), i.e., we set \(t_{\min} = t_1\), and hence compute \(\varsigma_1 = \ln(t_1)\) to be used in Eq. 15, where we used \(t_{1_k} = 10^{-3}\) hr by default.

For further details regarding computation of the computed \(p_c\) from Eqs. 8 or 13, we refer the readers to the papers by von Schroeter et al., Levitan, Onur et al., and Pimonov et al.

**Nonlinear History Matching for Pressure-Derivative-Rate Deconvolution**

Similar to the previous authors [von Schroeter et al. (2004), Levitan (2005), and Pimonov et al. (2009)], we will approach to deconvolution as a time-domain approach in which we seek a solution by history matching pressure-derivative data and rate data (if it is treated as unknown) using a Tikhonov type regularization (Tikhonov, 1963) on curvature of the desired solution. Before formulating the appropriate objective function to be used for pressure-derivative-rate deconvolution, we make a brief review and some important remarks on the use of pressure-derivative data in nonlinear history matching.

As mentioned previously, we focus on the use of pressure-derivative data for deconvolution. In the past, a number of authors (Barua et al. 1985; Carvalho et al. 1992; Rahon et al. 1997; Onur and Reynolds, 2002) have considered the use of pressure-derivative data for nonlinear parameter estimation based on pressure-derivative convolution (see Eq. 4 or 5) with a known reservoir model \(p_c(t)\) or \(g(t)\). However, all such work, except Onur and Reynolds, assume that the variance of pressure measurement errors and the “measurement errors” of pressure-derivative data are identical or diagonal. As shown by Onur and Reynolds, pressure-derivative data are not measured, but computed numerically as a linear combination of measured pressure data (e.g., see Eq. 9). As a result, the statistical model for the pressure measurement errors uniquely determines the error covariance matrix for pressure-derivative data. For instance, Onur and Reynolds show that if the measurement errors in pressure data can be modeled as independent normally distributed random variables with mean zero and a given variance (i.e., the error covariance matrix for pressure data is diagonal), the error covariance (or “weighting”) matrix to be used in history matching of pressure-derivative data will not be diagonal.

We let \(p'_m\) denote the \(N_p\)-dimensional vector of “measured” pressure-derivative data computed numerically at the \(N_p\) interior points, and let \(e'\) denote the vector of pressure-derivative data “measurement” errors. Throughout, \(N_d\) denotes the number of measured pressure-derivative data and equals to \(N_p\). Furthermore, we let \(p_m\) denote the \(N_p\)-dimensional vector of “measured” pressure data, and let \(e\) denote the vector of pressure data measurement errors.

Then, a stochastic model for the measured pressure-derivative data can be written as:

\[ p'_m = p'_m(z_d, q_m) + e', \]

(16)

where \(p'_m(z_d, q_m)\) denotes the \(N_p\)-dimensional vector of model pressure-derivative data computed by numerical differentiation of the model pressure data using the pressure-rate convolution equation (see Eq. 8 or 13 depending on the treatment of the first node). It should be pointed out that when \(q_m\) is uncertain and to be modified by deconvolution, then \(p'_m(z_d, q_m)\) becomes \(p'_m(z_d, q_c)\). The quantity \(z_d\) is the \(N\)-dimensional vector of response coefficients, and \(q_m\) and \(q_c\) are the \(N_q\)-dimensional vectors.
of measured and computed flow rate steps. In case of Eq. 13 is used, we will add $p_k(\tau_i)$ into the arguments of $p'_c$, i.e., $p'_c[z_d, q_m, p_k(\tau_i)]$.

As mentioned above, pressure-derivative are not measured, but are computed by numerical differentiation of measured pressure data using the three-point formula given by Eq. 9. As shown by Onur and Reynolds, Eq. 9 should be used for both measured and computed (or model) pressure data sets to avoid modeling error. Hence, the sets of measured and computed (or model) pressure-derivative data (Eq. 9) are linearly related to measured and computed (or model) pressure data by the following equations, respectively,

$$p'_m = Bp_m,$$  \hspace{1cm} (17)
and

$$p'_c = Bp_c,$$  \hspace{1cm} (18)

where $p_m$ denote the $N_p$-dimensional vector of computed (or model) pressure data and $p'_c$ is the $N_p$-dimensional vector of computed (or model) pressure-derivative data computed numerically at the $N_p$-2 interior points. In Eqs. 17 and 18, $B$ is an $N_p \times N_p$ matrix, referred to as the mapping matrix between pressure and pressure-derivative data, which is given explicitly as

$$B = \begin{bmatrix}
a_2 & b_2 & c_2 & 0 & \cdots & 0 \\
0 & a_3 & b_3 & c_3 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & a_{N_p-1} & b_{N_p-1} & c_{N_p-1}
\end{bmatrix}.$$  \hspace{1cm} (19)

Then, the covariance matrix for $e'$ is related to the covariance matrix for $e$ by the following equation (Onur and Reynolds, 2002):

$$C'_D = B C_D B^T,$$  \hspace{1cm} (20)

where $C'_D$ is the $N_p \times N_p$ covariance matrix for pressure-derivative measurement errors, and $C_D$ is the $N_p \times N_p$ covariance matrix for pressure measurement errors. Although Eq. 20 is quite general in the sense that the covariance matrix $C_D$ needs not to be diagonal, for simplicity, we will assume that $C_D$ is a diagonal matrix, i.e., we assume pressure measurement errors are uncorrelated, but each pressure point could have a different variance coming from a Gaussian distribution with mean zero. Note that even if $C_D$ is assumed to be diagonal, the derivative data covariance matrix $C'_D$ will not be a diagonal matrix. Onur and Reynolds present the conditions when $C'_D$ will be singular or nonsingular depending on the derivative data sets to be used in history matching. Therefore, for such details, we refer the readers to Onur and Reynolds’ paper.

The pressure-derivative data are computed from Eq. 9 at the $N_p-2$ interior points, and hence, the $C'_D$ will be nonsingular matrix and is given as (Onur and Reynolds 2002)

$$C'_D = \begin{bmatrix}
\sigma'_{11} & \cdots & \sigma'_{1(N_p-1)} & \sigma'_{1N_p} \\
\sigma'_{21} & \cdots & \sigma'_{2(N_p-1)} & \sigma'_{2N_p} \\
\vdots & \ddots & \vdots & \vdots \\
\sigma'_{(N_p-2)1} & \cdots & \sigma'_{(N_p-2)(N_p-1)} & \sigma'_{(N_p-2)N_p} \\
\sigma'_{(N_p-1)1} & \cdots & \sigma'_{(N_p-1)(N_p-1)} & \sigma'_{(N_p-1)N_p}
\end{bmatrix},$$  \hspace{1cm} (21)

where the elements of the matrix are given by

$$c'_{j-1,j-1} = c_j^2 \sigma^2_{p,j-1} + b_j^2 \sigma^2_{p,j} + a_j^2 \sigma^2_{p,j-1},$$  \hspace{1cm} (22)
and

$$c'_{j-1,j} = c_j b_j \sigma^2_{p,j-1} + b_j a_j \sigma^2_{p,j},$$  \hspace{1cm} (23)
and
\[ c'_{j-1,j+1} = c_j a_{j+2} \left( \sigma^2_p \right)_{j+1} \]  
\hspace{1cm} \text{(24)}

for \( j = 2, \ldots, N_p - 1 \). Note that \( C_p \) is a symmetric penta-diagonal matrix and thus elements \( c_{ij} = 0 \) if \( |i - j| > 2 \).

Now, we can propose the following objective function to reconstruct the response function coefficient matrix \( z_d \):
\[ O_q(z_d, q_c) = \frac{1}{2} \left[ \left( \left( C_p \right) \right)^{-1} [p'_c(z_d, q_c) - p'_m] + [q_c - q_m] [C_c]^{-1} [q_c - q_m] + [Dz_d - k] [C_c]^{-1} [Dz_d - k] \right] \]  
\hspace{1cm} \text{(25)}

In Eq. 25, \( C_q \) is the \( N_q \times N_q \) rate data covariance matrix to be assumed diagonal and is given as
\[ C_q = \sigma^2_q \begin{bmatrix} 1/w^2_{q,1} & 0 & 0 & \cdots & 0 \\ 0 & 1/w^2_{q,2} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 1/w^2_{N_q,N_q} & 0 \\ 0 & \cdots & \cdots & 0 & 1/w^2_{N_q,N_q} \end{bmatrix} \]  
\hspace{1cm} \text{(26)}

where \( w_{q,j} \) are the weights having the same functionality as in the paper of Pimonov et al. (2009), and \( \sigma^2_q \) is the variance of rate errors, which are assumed to be Gaussian with zero mean. Note that in cases which flow-rate steps are not treated as unknown, then \( q_c \) is eliminated from the arguments of the objective function, and the second flow-rate mismatch term in the right-hand side of Eq. 25 is deleted. Also, \( q_c \), in the arguments of \( p'_c(z_d, q_c) \) in the right-hand-side of Eq. 25 will be replaced by \( q_m \). \( D \) is the \( (N-1) \times N \) curvature measure matrix, where \( N \) is the number of logarithmically equally spaced nodes used for reconstruction of \( z_d \) function, with the elements for the first row \( (i = 1) \) and the other rows \( (i = 2, 3, \ldots, N-1) \) for a logarithmically uniformly spaced nodes, given as
\[ d_{i,j} = \begin{cases} -1/\Delta \varsigma, & \text{for } j = 1 \\ 1/\Delta \varsigma, & \text{for } j = 2, \ldots, N-1 \\ 0, & \text{for } j = 3, 4, \ldots, N \end{cases} \]  
\hspace{1cm} \text{(27)}

and
\[ d_{i,j} = \begin{cases} 1/\Delta \varsigma, & \text{for } j = i-1 \\ -2/\Delta \varsigma, & \text{for } j = i \\ 1/\Delta \varsigma, & \text{for } j = i+1 \\ 0, & \text{otherwise} \end{cases} \]  
\hspace{1cm} \text{(28)}

where \( \Delta \varsigma = \varsigma_{k+1} - \varsigma_k, \ k = 1, 2, \ldots, N-1 \), is the node increment of logarithmically equally spaced grid \( \varsigma_k \), and \( k \) is an \( N-1 \) dimensional vector given as \( \mathbf{k} = (1, 0, \ldots, 0)^T \) [see von Schroeter et al. (2004)]. \( C_c \) is an \( (N-1) \times (N-1) \) diagonal curvature covariance matrix given as
\[ C_c = \sigma^2_c \mathbf{I}, \]  
\hspace{1cm} \text{(29)}

where \( \mathbf{I} \) is the identity matrix, and \( \sigma^2_c \) represents the “variance” of the curvature constraint and is usually taken as 0.05. The value of \( \sigma^2_c \) should provide a small degree of regularization and at the same time should not over constrain the problem and create a significant bias. We have found that the value \( \sigma_c = 0.05 \) seems to provide acceptable levels of response smoothness; however, it sometimes needs to be changed for a more suitable value in an ad-hoc manner during the calculations.
The objective function given by Eq. 25 is nonlinear with respect to the model parameters, $z_d, q_c$, and $p_u(\tilde{\tau}_i)$ in case if we use the deconvolution equation of Levitan, see Eq. 13 and can be minimized effectively by using the Levenberg-Marquardt method with a restricted step procedure (Fletcher 1987). The initial parameter values of $z_d, q_c$, and $p_u(\tilde{\tau}_i)$ in case of Eq. 13 to be used in minimizing Eq. 25 can be selected as suggested by von Schroeter et al.

Once the $z_d$-deconvolved responses are determined by minimization of Eq. 27, the unit-rate drawdown response $p_u(\tau)$ and its logarithmic derivative $dp_u/d\ln \tau$ at the nodes $\tilde{\tau}_k$ are computed, respectively, from

$$p_u(\tilde{\tau}_k) = e^{(z_d) \ln \tau}; \text{for } k = 1, \ldots, N,$$

$$p_u(\tilde{\tau}_k) = e^{(z_d) \ln \tau} + \sum_{j=2}^{k} \frac{e^{(z_d) \ln \tau} - e^{(z_d) \ln \tau}}{\ln \tilde{\tau}_j / \ln \tilde{\tau}_{j-1}}; \text{for } k = 2, \ldots, N,$$

$$\frac{dp_u}{d\ln \tau}(\tilde{\tau}_k) = e^{(z_d) \ln \tau}; \text{for } k = 1, \ldots, N.$$

In case where Eq. 13 is used to obtain the computed pressure in Eq. 25, we replace $e^{(z_d) \ln \tau}$ by $p_u(\tilde{\tau}_i)$ (to be estimated by minimization of the objective function) in Eqs. 30 and 31.

Simultaneous History Matching of Pressure and Pressure-Derivative Data
One may question whether simultaneously matching both pressure and pressure-derivative data could provide an advantage in reconstruction of the deconvolved unit-rate drawdown response. In fact, a similar idea was investigated by Onur and Reynolds (2002) in the context of information content of pressure and pressure-derivative data for nonlinear parameter estimation with a known well/reservoir model. Based on Onur and Reynolds work, we can show that if the proper data covariance matrix incorporating both pressure and pressure-derivative data, e.g., a matching data set including all interior pressure-derivative data plus two appropriate pressure points, is used, then the objective function to be used for reconstructing the unit-rate drawdown response will be identical to the objective function that would be obtained by regression on pressure data only and hence, will yield the same estimates of the unit-rate response and confidence intervals that would be obtained by regression on pressure data only.

Onur and Reynolds show that to obtain a nonsingular positive-definite data error covariance matrix to be used in history matching pressure and pressure-derivative data simultaneously, the mapping matrix between the pressure data set and given pressure-derivative data set should be a nonsingular square matrix. One of the appropriate data sets satisfying this condition is the one that contain pressure-derivative data at $N_p-2$ interior points plus the first two pressures. For this data set, let $(p_{m})_{prd}$ is the $N_p$-dimensional vector of measured data, defined as

$$(p_{m})_{prd} = \left[(p_m)_1, (p_m)_2, (p_m)_3, \ldots, (p_m)_{N_p-1}\right]^T,$$

and the $N_p$-dimensional vector $(p_{c})_{prd}$ of the model data is defined similarly. Further, let $B_{prd}$ and $(C_D)_{prd}$ denote the mapping matrix and the data covariance matrix associated with this data set. The mapping matrix $B_{prd}$ is an $N_p \times N_p$ square matrix given as

$$B_{prd} = \begin{bmatrix}
1 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & 1 & 0 & \cdots & \cdots & \cdots & 0 \\
a_2 & b_2 & c_2 & 0 & \cdots & \cdots & \cdots \\
0 & a_3 & b_3 & c_3 & 0 & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & a_{N_p-1} & b_{N_p-1} & c_{N_p-1} & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & 0 & a_{N_p-1} & b_{N_p-1} & c_{N_p-1}
\end{bmatrix}.$$  

$B_{prd}$ is a lower triangular matrix with nonzero diagonal elements because all the $c_j$ (Eq. 12) are nonzero, and thus, it is nonsingular (Strang 1988). Note that the mapping matrix relates the set $(p_{m})_{prd}$ given by Eq. 33 to the measured pressure set $p_m$, and the corresponding model set $(p_{c})_{prd}$ to the model pressure set $p_c$ by the following relations, respectively:

$$(p_{m})_{prd} = B_{prd}p_m,$$
and

$$\langle p_c \rangle_{p+d} = B_{p+d} P_c.$$  \hfill (36)

The data covariance matrix $\langle C_D \rangle_{p+d}$ is also an $N_p \times N_p$ square matrix and is related to the $N_p \times N_p$ data covariance matrix of pressure measurements $C_D$ and the $N_p \times N_p$ mapping matrix $B_{p+d}$ as

$$\langle C_D \rangle_{p+d} = B_{p+d} C_D B_{p+d}^T.$$  \hfill (37)

The data covariance matrix $\langle C_D \rangle_{p+d}$ is nonsingular because $B_{p+d}$ and $C_D$ are nonsingular. The matrix $\langle C_D \rangle_{p+d}$ is explicitly given as

$$\langle C_D \rangle_{p+d} = \begin{bmatrix}
\sigma_{1,1}' & 0 & \cdots & 0 & \\
0 & \sigma_{2,2}' & \cdots & 0 & \\
\vdots & \vdots & \ddots & \vdots & \\
0 & 0 & \cdots & \sigma_{N_p, N_p}'
\end{bmatrix},$$  \hfill (38)

where

$$c'_{j,j} = \sigma_p^2_j, \text{ for } j = 1, 2,$$  \hfill (39)

$$c'_{j,j+1} = b_j \sigma_p^2_j, \text{ for } j = 2,$$  \hfill (40)

$$c'_{j,j+2} = a_{j+1} \sigma_p^2_j, \text{ for } j = 1, 2,$$  \hfill (41)

$$c'_{j,j} = a_{j-1} \sigma_p^2_j + h^{2}_{j-1} \left( \sigma_p^2_{j-1} \right), \text{ for } j = 3, \ldots, N_p,$$  \hfill (42)

$$c'_{j,j+1} = a_j b_{j-1} \left( \sigma_p^2_{j-1} \right) + b_j c_{j-1} \left( \sigma_p^2_j \right), \text{ for } j = 3, \ldots, N_p - 1,$$  \hfill (43)

and

$$c'_{j,j+2} = a_{j+1} c_{j-1} \left( \sigma_p^2_j \right), \text{ for } j = 3, \ldots, N_p - 2.$$  \hfill (44)

Eqs. 39-44 are sufficient to define the entire matrix of $\langle C_D \rangle_{p+d}$ because it is symmetric.

Now, we can propose the following objective function to reconstruct the unit-rate response function $z$:

$$O_{p+d}(z_d, q_c) = \frac{1}{2} \left[ \left( \langle p_c \rangle_{p+d} \langle z_d, q_c \rangle - \langle p_m \rangle_{p+d} \right) \left( \langle C_D \rangle_{p+d} \right)^{-1} \left[ \langle p_c \rangle_{p+d} \langle z_d, q_c \rangle - \langle p_m \rangle_{p+d} \right]^T + \frac{1}{2} [q_c - q_m]^T \left( C_q \right)^{-1} [q_c - q_m] + \frac{1}{2} [Dz_d - k]^T \left( C_c \right)^{-1} [Dz_d - k] \right].$$  \hfill (45)

Using Eqs. 35-37 in Eq. 45 and performing the algebra (we skip details) to simplify the resulting equation, it can be shown that

$$O_{p+d}(z_d, q_c) = \frac{1}{2} \left[ p_c^T [\langle z_d, q_c \rangle - \langle p_m \rangle] C_D^{-1} [p_c^T [\langle z_d, q_c \rangle - \langle p_m \rangle] + \frac{1}{2} [q_c - q_m]^T \left( C_q \right)^{-1} [q_c - q_m] + \frac{1}{2} [Dz_d - k]^T \left( C_c \right)^{-1} [Dz_d - k] = O_p(z_d, q_c) \right.$$

$$+ \frac{1}{2} [Dz_d - k]^T \left( C_c \right)^{-1} [Dz_d - k] = O_p(z_d, q_c).$$  \hfill (46)
where \( O_p(z_d, q_d) \) defines the objective function to be minimized when a data set containing \( N_p \) pressures is used to reconstruct the \( z_d \) response. Eq. 46 is essentially the same objective function used by von Schroeter et al., Levitan, and Pimonov et al. This result clearly indicates that adding pressure into pressure-derivative data set will not provide an advantage because the addition of pressure and pressure-derivative data with the appropriate data covariance matrix will yield the same objective functions and hence the same estimates of the response vector \( z_d \) and the flow rate vector \( q_d \), and their associated confidence intervals. However, we should note that adding pressure data into reconstruction of the response function \( z_d \) introduces a dependency to the initial reservoir pressure \( p_0 \), and hence the information content of pressure data is more than that of using only pressure-derivative data in the objective function given by Eq. 25. In fact, this was our main objective in this work because we wanted to develop a deconvolution method that could be used to reconstruct the response function \( z_d \) without the requirement of the initial reservoir pressure.

Some Implementation Aspects

Regarding the computation of the pressure-derivative data covariance matrix \( C'_{Dp} \) given by Eq. 21 and its inverse \((C'_{Dp})^{-1}\) in Eq. 25, we would like to make a few comments to highlight a few subtle points.

If deconvolution is applied to all flow and buildup periods of the same test sequence, pressure-derivative data \( p'_j \) are computed by using Eq. 9 at the \( N_p \)-2 interior time points based on the total times \( t_j, j=2,3,\ldots,N_p-1 \), i.e.,

\[
p'_j = \frac{dp}{d\ln t_j} = a_j p_{j-1} + b_j p_j + c_j p_{j+1},
\]

with the coefficients computed from Eqs. 10-12. Once we compute the coefficients \( a_j, b_j, \) and \( c_j \), we can compute the elements of the derivative data covariance matrix \( C'_{Dp} \) using Eqs. 22-24 with given values of the variance of each pressure point, i.e., \( \sigma^2_{p,j} \). If the number of pressure points is large, then storing \( C'_{Dp} \) may be difficult, but then we can use special storage schemes so that only nonzero elements are stored, such as the so-called coordinate format, compressed sparse row, or compressed column storage scheme (Saad 2003). We have used compressed row storage scheme. However, one can use filtering techniques to reduce the number of pressure points so that we can have manageable sizes of pressure points and the associated pressure-derivative covariance matrix.

Our implementation, when minimizing the objective function given by Eq. 25, does not require the direct computation of the inverse of the penta-diagonal covariance matrix \( C'_{Dp} \) because we use a LU decomposition of \( C'_{Dp} \) based on a-matrix-by-vector operation to efficiently compute the derivative mismatch term given as the first term in the right-hand side of Eq. 25. Although we have not considered in our implementation, for large sized \( C'_{Dp} \) matrix, one can use iterative methods for computing the derivative mismatch term (see van der Vorst 2003).

In case where we will apply deconvolution to individual flow or buildup period pressure points (note that this case will be the most practical case to consider because most of the time, we have inconsistent data set with the convolution equation to work with as mentioned previously), we can compute the pressure-derivative using Eq. 47 at the \( N_p \)-2 interior time points based on the total times \( t_j, j=2,3,\ldots,N_p-1 \) and use these derivative data in history matching. We can also consider history matching pressure-derivative obtained by differentiation with respect to the elapsed times \( (\Delta t_j) \) for the considered individual period. Here, the elapsed times are computed using the starting (total) time of the individual period. It can be shown that the pressure-derivative data with respect to elapsed times could easily be computed from the values of pressure-derivative data with respect to total times by using the following equation:

\[
\frac{dp}{d\ln \Delta t_j} = \frac{\Delta t_j}{t_j} \frac{dp}{d\ln t_j},
\]

and the coefficient \( a_j, b_j, c_j \) could be computed from Eqs. 10-12 with \( t_j \) replaced by \( \Delta t_j \). Then, we can compute the elements of the matrix \( C'_{Dp} \) using Eqs. 22-24.

Regarding smoothing pressure-derivative data, we have made a detailed investigation of the effect that smoothing derivative has on the estimated response coefficients \( z_d \). We found that smoothing has no advantages if derivative data are to be used in history matching and the noise in pressure data follows a Gaussian distribution with mean zero and specified variance. However, this does not mean that we should include outlier pressure points or inaccurate pressure measurements related to gauge performance when computing pressure derivatives. Such points should be eliminated before applying deconvolution based on derivative data.

Example Applications

To test the new pressure-derivative deconvolution method, we consider one synthetic (simulated) test and two field test examples. Pressure and rate data for the synthetic example were by the Ecrin software (2009). In all applications to be given, we have used our implementation of the method based on the unit-slope assumption at the first node.
Synthetic Test Example. We consider a simulated well-test example (see Fig. 1 for pressure/rate data) for a vertical well with wellbore-storage effects in a closed homogeneous, isotropic reservoir (see Table 1 for input model parameters and Fig. 2 for the well/reservoir geometry). Pressure data were generated by assuming a gauge with a noise of 0.01 psi, i.e., $\sigma_{p_j} = 0.01 \text{ psi}$, $j = 1, 2, \ldots, N_p$, and rate history does not contain any errors. The data set is consistent with the deconvolution model of Eq. 1. The number of pressure data points is $N_p = 458$, and the total duration of the test is 300 hr.

Figure 3 shows the absolute value of pressure-derivative data, computed by the use of Eq. 9, to be used in history matching based on the objective function given by Eq. 25. No smoothing has been applied to the data shown in Fig. 3. As pressure-derivative data are negative for drawdown periods, we have plotted the absolute values of the pressure-derivative on vertical log scale of Fig. 3, but in history matching, both negative and positive pressure derivative data are used. The total number of pressure-derivative data to be used in history matching is 456, i.e., $N_d = N_p - 2 = 456$.

| TABLE 1—INPUT PARAMETERS FOR SYNTHETIC EXAMPLE |
|-----------------|-----------------|
| $\phi$ (fraction) | 0.10            |
| $h$ (ft)         | 30              |
| $c_i$ (psi$^{-1}$) | $3.0 \times 10^{-5}$ |
| $\mu$ (cp)      | 1.0             |
| $r_w$ (ft)       | 0.354           |
| $S$ (dimensionless) | 2.0            |
| $C_w$ (bbl/psi)  | 0.005           |
| $k$ (md)         | 100             |
| $p_0$ (psi)      | 5000            |
| Geometry         | See Fig. 2      |
| L1 – No flow (ft) | 500             |
| L2 – No flow (ft) | 500             |
| L3 – No flow (ft) | 1500            |
| L4 – No flow (ft) | 1500            |

As this is a consistent data set, we have used all the pressure-derivative from 0.002 to 299.21 hr (see Fig. 3). We treat flow rate as fixed in minimization; i.e., no regression on rate data, and set the standard deviation of the curvature constraint equal to $\sigma_c = 0.05$. We use 70 logarithmically equally spaced nodes to reconstruct the unit-rate responses.

Figure 4 presents the deconvolved unit-rate responses $p_u(t)$ and $p_u'(t)$, in comparison with the true unit-rate drawdown responses, $p_u(t)$ and $p_u'(t)$ generated using the true well/reservoir model. The match of computed pressure-derivative data with the “measured” pressure-derivative data is shown in Fig. 3. The overall Root-Mean-Square (RMS) error for the match is about 1184 psi. As can be seen from Figs. 3 and 4, the agreement between the true and reconstructed unit-rate drawdown responses and the match of measured pressure-derivative with the computed pressure-derivative are excellent.

Although not shown here, we have also considered reconstruction of the unit-rate responses by using the pressure-derivative data only for the last buildup period, in the time interval from 180 to 300 hr with the total number of derivative points equal to $N_d = 143$ points. The results obtained were as perfect as those shown in Fig. 4, which were obtained by deconvolution of entire pressure-derivative data of 456 points, without using an initial reservoir pressure.

All these results show that our deconvolution method works well for the synthetic example. Next, we will investigate the applicability of the method on two field examples.

Field Test Example 1. Here, the pressure and rate data from an actual drillstem test (DST), as shown in Fig. 5, will be used for deconvolution. The total duration of the test is about 49 hr. The test sequence contains two drawdown and two buildup periods. The duration of the first drawdown is short; it is about 0.75 hr, the duration of the second drawdown is about 11 hr. The duration of the first buildup (BU1) is about 22 hr, whereas the duration of the second buildup (BU2) is about 15 hr. The pressure data for both drawdown periods are noisy. Therefore, we will apply deconvolution to individual buildup periods. Before the DST test, a wireline formation test (WFT) was conducted for the same zone, and the initial reservoir pressure determined from this WFT was 5229.212 psi at a common datum. However, due to a possible uncertainty in the initial reservoir pressure, we will investigate both using Levitan et al.’s suggested trial-and-error procedure and the new derivative deconvolution method.
Figure 1— Pressure and rate data for Synthetic Test Example.

Figure 2— Well/reservoir configuration for Synthetic Test Example.

Figure 3— “Measured” vs. computed pressure-derivative data for Synthetic Test Example.
Figure 4—Deconvolved unit-rate drawdown responses for Synthetic Test Example data in Fig. 1, in comparison with the associated true unit-rate drawdown responses generated from the model.

Figure 5—Pressure and rate data for Field Test Example 1.

Figure 6 presents the conventional log-log diagnostic plots of the rate-normalized pressure change and pressure-derivative data based on the multi-rate superposition time plotted versus elapsed time for BU1 and BU2 periods. Here, pressure change and pressure-derivative data were normalized by the flow rates prior to the associated buildup periods. From Fig. 6, we observe that both BU exhibit a radial flow in the time interval from 0.01 to 0.1 hr, but the radial flow slope for BU2 is slightly lower than that of BU1. This may indicate an increase in mobility from BU1 to BU2. After 0.1 hr, both buildups exhibit a similar up trend due to boundary effects. However, we think that late-portions of the BU1 and BU2 data suffer from “residual” superposition effects due to the use of multi-rate superposition time based on the well-known logarithmic approximation. Such an approximate multi-rate superposition time often distorts the late-time portions of the buildup derivative data if the duration of the preceding production period is short, and the effects of the boundaries are felt at the well response, in turn which causes a deviation from the equivalent drawdown derivative response. This will be later verified by comparing deconvolved responses with the conventional responses shown in Fig. 6.

It is also important to note the pressure-derivative for BU1 fluctuates severely after 10 hr. It is unlikely that this is a reservoir behavior. Therefore, we will assign very small weights or equivalently very large error variances (e.g., $\sigma_i^2 = 1 \times 10^{10}$) for pressure measurements after 10 hr, when performing deconvolution.
Next, we apply the new derivative deconvolution method to reconstruct the unit-rate drawdown responses by history matching the measured pressure-derivative data for BU1 and BU2 periods. BU1 and BU2 pressure-derivative data to be used in history matching based on the objective function given by Eq. 25 are shown in Fig. 7. Again we should note that no smoothing was applied when computing the buildup pressure-derivative data shown in Fig. 7.

Deconvolved responses for the unit-rate drawdown responses based on the pressure-derivative data for both buildup periods are shown in Fig. 8. The matches of the computed pressure-derivative data by deconvolution with the measured pressure-derivative data are shown in Fig. 7. For both deconvolution applications we have used the same variance of curvature constraint equal to $\sigma_c^2 = 0.05^2$ and the same error variances of pressure measurements equal to $\langle \sigma_p^2 \rangle = 0.03^2$ psi$^2$. As mentioned above, the pressure-derivative after 10 hr of elapsed times may not be representative of the reservoir behavior for the BU1 period, and hence, we used a large variance, e.g., $\langle \sigma_p^2 \rangle = 1 \times 10^8$, for elapsed times greater than 10 hr.

Figure 9 shows the deconvolved responses obtained by using individual buildup pressures with an initial reservoir pressure value of 5229.212 psi. For both buildup periods, we have used the same values of $\sigma_c^2 = 0.05^2$ and $\langle \sigma_p^2 \rangle = 0.03^2$ for reconstructing the deconvolved responses shown in Fig. 9. In Fig. 9, for comparison purposes, we have also plotted the deconvolved responses reconstructed from BU2 pressure-derivative data. Clearly, all deconvolved responses are in excellent agreement with the measured data.
agreement. This also verifies the validity of the initial reservoir pressure of 5229.212 (converted to the same datum of DST) obtained from a WFT gauge.

Finally, we present comparison of conventional buildup responses given in Fig. 6 with the deconvolved unit-responses based on the pressure-derivative data given in Fig. 8 to delineate the differences among these responses. This comparison is shown in Fig. 10. Clearly, conventional buildup responses show different behavior from unit-rate responses at late times, and also span shorter time periods (limited to the durations of the buildup periods) than the unit-rate responses, as expected.

Figure 8– Deconvolved responses from BU1 and BU2 pressure-derivative in Fig. 7 for Field Test Example 1.

Figure 9– Deconvolved responses from BU1 and BU2 pressure data with $p_0 = 5229.21$ psi, in comparison with the deconvolved responses from BU2 pressure-derivative data, Field Test Example 1.
0.001 0.01 0.1 1 10 100

Unit-rate constant drawdown responses, and conventional buildup responses, psi/STB/D)

Figure 10– Comparison of deconvolved responses from BU1 and BU2 pressure-derivative data with conventional buildup responses, Field Test Example 1.

**Field Test Example 2.** This example (Kuchuk et al. 2010) consists of an extended drawdown and a short buildup test sequence taken over a 1346-hr period from a horizontal well. The rate measurements were acquired every 2 hours at the surface. As can be seen from Fig. 11, both pressure and rate decline steadily during the production period that followed a 270-hr buildup test. The initial reservoir pressure is not known for this example. Also the reservoir model is quite complicated to apply Horner method to obtain the initial reservoir pressure. Therefore, for this data set, unlike the Field Test Example 1, we cannot apply the procedure of Levitan et al. (2006) because the buildup data in this example has insufficient information to accurately determine the initial pressure and unit-rate drawdown responses as his deconvolution method requires that we determine the initial pressure from at least two consistent buildup periods.

In this example, we will use the pressure-derivative data (shown in Fig. 12) computed for the entire duration of the test, and treat flow rate (actually a piecewise constant representation of the measured flow rate) history as unknown in deconvolution. Note that we have plotted the absolute values of the pressure-derivative data in Fig. 12 because pressure-derivative data are negative during most of the production period.
Kuchuk et al. (2010) have considered in detail the deconvolution analysis of the same data set, but using the deconvolution algorithm of Pimonov et al. (2009) based on history matching of pressure data. They have regressed on both entire pressures and measured flow rate data to estimate the unknown initial reservoir pressure, unit-rate responses, and the flow rate history, simultaneously. Here, we will use their best estimated (or “optimal”) algorithmic parameters; $\sigma_i^2 = 0.05^2$, $\sigma_q^2 = 17.8^2$, and $\sigma_p^2 = 1.41^2$ in deconvolution based on history matching pressure-derivative data of Fig. 12. In our application, only the unit-rate drawdown response and flow-rate history are to be estimated by deconvolution by history matching pressure-derivative and measured flow-rate data because our method is free of initial reservoir pressure.

Figure 12– Measured vs. computed pressure-derivative data for Field Test Example 2.

Figure 13 presents deconvolved unit-rate drawdown responses in comparison with conventional buildup as well as the deconvolved responses obtained by Kuchuk et al. (2010) with initial reservoir pressure as unknown. The pressure-derivative data computed by deconvolution are compared with the measured pressure-derivative data in Fig. 12. The match of pressure-derivative data is quite good – the overall Root-Mean-Square (RMS) error for derivative match is about 13000 psi, but this is expected because the magnitude of errors in derivative data is usually much larger than that of errors in pressure data. Also some of the spikes seen in computed derivative data are due to our piecewise approximation of the flow rate history. We have used 89 piecewise constant flow-rate steps to represent measured flow rate data. Perhaps, we could have used less flow rate steps to represent measured rate to reduce the magnitudes of some of the spikes in derivative data shown in Fig. 12. Nevertheless, the agreement between deconvolved responses obtained by deconvolution based on pressure-derivative and pressure matching (Kuchuk et al.) are excellent. Note that Kuchuk et al. estimated the initial reservoir pressure by deconvolution to be about 4791.41 psi. We have obtained almost the identical deconvolved responses without initial reservoir pressure.

Like the von Schroeter et al. and Pimonov et al. deconvolution algorithms, our deconvolution method also gives the user an option to adjust the flow rate history in deconvolution, as we did in this example application, and to compute the 95% confidence intervals for the estimated unit-rate and flow rate steps (if treated as unknown in deconvolution). For example, Fig. 14 shows the estimated flow rate steps and their associated 95% confidence intervals in comparison with the measured flow rate history. The rate match seems reasonably good because rate for each flow period is not changed by more than 10% – the overall RMS error for rate is 178 STB/D (Fig. 14). Also 95% confidence intervals for estimated flow-rate steps are quite narrow (with a maximum of ± 70 STB/D) so that one cannot readily see the lower and upper limits of 95% confidence intervals on the vertical scale of Fig. 14.
Conclusions

A new deconvolution method based on pressure derivative is presented for the interpretation of pressure transient pressure and flow rate data. The method removes the dependency of the deconvolved constant-rate drawdown responses to the initial reservoir pressure. The method is based on the weighted nonlinear least-squares objective function using pressure-derivative data with the appropriate data error covariance matrix. Similar to Pimonov et al. method, our method also allows considering errors with different magnitudes in various sections of pressure and rate data by incorporating variable variances in the objective function and thus it can process such data to estimate the unit-rate drawdown (or impulse) response of the system, without the knowledge of the initial reservoir pressure. Additionally flow rates can be adjusted by our method.

The new method was successfully applied to one synthetic and two field pressure transient data sets; one DST and one horizontal well test data. Overall, the method was proved to be robust and useful for different practical applications of deconvolution analysis, and, most importantly for avoiding an incorrect interpretation due to an error or uncertainty in the initial reservoir pressure.
Acknowledgements
The authors would like to express their gratitude to Istanbul Technical University, Department of Petroleum and Natural Gas Engineering, and Schlumberger for permission to publish this paper. To generate pressure and rate data for the synthetic test example 1 application, we have used Ecrin from Kappa Engineering. The first author thanks Kappa Engineering for providing Ecrin free for educational and research purposes for the Department of Petroleum and Natural Gas Engineering at ITU. The authors extend their special thanks to Dr. Gilvan Soares Feitosa of Petrobras for providing the data for Field Test Example 1.

Nomenclature

\( a_j \) = coefficient defined by Eq. 10

\( b_j \) = coefficient defined by Eq. 11

\( B \) = mapping matrix defined by Eqs. 19 and 36

\( c \) = isothermal compressibility, \( \text{L}^2/\text{m}, 1/\text{psi}, [1/\text{atm}] \)

\( c_j \) = coefficient defined by Eq. 12

\( c' \) = elements of pressure-derivative data covariance matrix \( C'_D \)

\( C_w \) = wellbore storage coefficient, \( \text{L}^4/\text{t}, \text{stb}/\text{psi} \ [\text{cm}^3/\text{atm}] \)

\( C_D \) = pressure data error covariance matrix

\( C_c \) = covariance matrix for curvature constraints

\( C_q \) = rate data covariance matrix

\( D \) = pressure-derivative data covariance matrix

\( g(t) \) = unit-rate impulse function, \( g(t) = dp_a(t)/dt, \text{m}/\text{L}^4\text{t}, \text{psi}/\text{bbl} \ [\text{atm/cm}^3] \)

\( h \) = thickness, \( \text{L}, \text{ft}, [\text{cm}] \)

\( I \) = identity matrix

\( k \) = permeability, \( \text{L}^2, \text{md} \ [\text{Darcy}] \)

\( k \) = vector in curvature measure

\( L \) = distance to no-flow boundary, \( \text{L}, \text{ft}, [\text{cm}] \)

\( N \) = number of nodes for the deconvolved response

\( N_d \) = number of pressure-derivative data points to be history matched

\( N_p \) = number of pressure data points to be history matched

\( N_q \) = number of unknown flow rates

\( p \) = pressure, \( \text{m}/\text{L}^2, \text{psi} \ [\text{atm}] \)

\( p' \) = vector of pressures

\( p' \) = vector of pressure-derivative data

\( p_0 \) = initial reservoir pressure, \( \text{m}/\text{L}^2, \text{psi} \ [\text{atm}] \)

\( p_w \) = unit-rate drawdown-pressure response, \( \text{m}/\text{L}^4\text{t}, \text{psi}/\text{bbl}/\text{d} \ [\text{atm}/(\text{cm}^3/\text{s})] \)

\( q \) = flow rate, \( \text{L}^3/\text{t}, \text{bbl}/\text{day} \ [\text{cm}^3/\text{s}] \)

\( r_w \) = wellbore radius, \( \text{L}, \text{ft} \ [\text{cm}] \)

\( q \) = vector of flow rates

\( S \) = skin factor, dimensionless

\( t \) = time, \( \text{t}, \text{hour} \ [\text{s}] \)

\( \Delta t \) = elapsed time, \( \text{t}, \text{hour} \ [\text{s}] \)

\( T \) = total duration of pressure/rate measurements, \( \text{t}, \text{hour} \ [\text{s}] \)

\( w \) = weights for flow rate data, dimensionless

\( z_d \) = response function

\( (z_d) \) = response coefficients

\( z_d \) = vector of response coefficients

\( \alpha_j \) = time of start for \( q_j \) flow rate step, \( \text{t}, \text{hour} \ [\text{s}] \)

\( \beta_j \) = time of end for \( q_j \) flow rate step, \( \text{t}, \text{hour} \ [\text{s}] \)

\( \mu \) = viscosity of fluid, \( \text{m}/\text{L}t, \text{cp} \ [\text{cp}] \)

\( \phi \) = porosity, fraction

\( \sigma \) = standard deviation of errors

\( \tau \) = variable of integration

\( \tilde{\tau} \) = time nodes where \( z_d \) response function to be reconstructed

\( \varsigma \) = variable of integration based on natural logarithm of time

Subscripts

\( c \) = curvature or computed

\( d \) = deconvolved
\[ m = \text{measured} \]
\[ \text{min} = \text{minimum} \]
\[ p = \text{pressure} \]
\[ p+d = \text{pressure plus derivative} \]
\[ q = \text{rate} \]
\[ u = \text{unit-rate} \]

**Superscripts**

\[ T = \text{transpose of a matrix or vector} \]
\[ ^{-1} = \text{inverse of a matrix} \]
\[ ^\prime = \text{differentiation with respect to natural logarithm of time} \]

**SI Metric Conversion Factors**

- bbl \( \times 1.589873 \ \text{E}^{-01} = m^3 \)
- ft \( \times 3.048 \ \ast \ \text{E}^{-01} = m \)
- psi \( \times 6.894757 \ \text{E}^{+00} = \text{kPa} \)

*Conversion factor is exact.

**References**


