Tu-04-10

Crossline Reconstruction Using Aliased 3D Deghosted Up- and Downgoing Wavefields

D.-J. van Manen* (Schlumberger), M. Vassallo (WesternGeco), A.K. Özdemir (WesternGeco), A. Özbek (Schlumberger) & J.O.A. Robertsson (ETH Zürich)

SUMMARY

This paper formulates a new model for crossline reconstruction that relies on aliased, 3D decomposed, up- and downgoing waves. It also presents a closed form approach to solve the joint reconstruction and 3D deghosting problem. The new model provides additional insight into how the ghost reflection and the vertical component of particle velocity/acceleration are exploited in crossline reconstruction and joint 3D deghosting solutions for towed streamer applications.
Introduction

It is well known that the particle acceleration recorded by a multi-component streamer is directly proportional to the gradient of the pressure wavefield. Robertsson et al. (2008) showed how this can be used to reconstruct the wavefield in between streamers and double the Nyquist wavenumber, exploiting a generalization of classical sampling theory. Vassallo et al. (2010) showed that under mild conditions, data-dependent matching pursuit techniques can go significantly beyond this and reconstruct even data subject to high-order aliasing.

On the other hand, Robertsson et al. (2009) proposed a method that decomposes the vertical component of particle velocity at the streamer locations correctly into up- and downgoing waves in full 3D. The key observation is that only the pressure needs to be spatially filtered and this can be done correctly once the pressure is reconstructed in the crossline direction. Thus, it is clear how reconstruction with horizontal gradient measurements can provide the total wavefield (i.e. sum of the up and downgoing waves) in between the streamers and how, in turn, this can be used to give 3D decomposed data at the streamer locations.

To address the question of how to obtain the 3D deghosted wavefield in between the streamers, Özbek et al. (2010) introduced the concept of joint interpolation and deghosting and proposed a matching pursuit technique that simultaneously matches the pressure, crossline, and vertical components of particle acceleration measured at the streamer locations. Candidate basis functions for the upgoing wavefield are ghosted with the theoretical ghost operators for the different components and the optimal amplitude and phase found for a range of crossline wavenumbers by minimizing the residual between the ghosted basis functions and the observations in an iterative scheme. The method called Generalized Matching Pursuit (GMP) also has been found to reconstruct data subject to very high orders of aliasing.

The role of both the ghost model and the vertical component in the crossline reconstruction and joint deghosting process is not entirely intuitive. Also, it may not be immediately obvious that the multi-component streamer data contain enough information to achieve joint interpolation and deghosting. Özdemir et al. (2010) derive a generalization of classical sampling theory they call Optimal Deghosting and Reconstruction (ODR) that addresses such questions to a large extent. Here we introduce another model for the crossline reconstruction based on aliased, 3D deghosted, up- and downgoing waves and show explicitly that the data contain enough information to jointly interpolate and deghost the wavefield provided a model for reflection at the free surface is used. The stepwise approach may allow a somewhat more rigorous theoretical treatment, detailed analysis and physical insight.

Theory and Method

We start by noting that in the 3D decomposition method by Robertsson et al. (2009) outlined above no information about the free-surface was used. This is characteristic of wavefield decomposition algorithms and contrasts with deghosting algorithms that require the receiver depth as an input parameter. On the other hand, we know that the up- and downgoing wavefields are related through reflection at the free-surface and we are going to exploit this. If $z_r$ denotes the depth of the streamer below the free-surface and if we assume that the source depth, $z_s$, is larger than the streamer depth or, equivalently, that the direct wave has been removed from the data, then the relation between the up- and downgoing vertical component wavefields can be written (in the frequency, inline-, and crossline wavenumber domain) as:

$$V_z^+(f, k_x, k_y) = \exp(i2k_z z_r) V_z^-(f, k_x, k_y)$$

(1)

Where $k_z = \sqrt{(\omega/c)^2 - k_x^2 - k_y^2}$. The verification of equation (1) requires the particle velocity to be sufficiently densely sampled in the t, x, and y directions. However, as stated above, the data are aliased in the crossline direction.
One of the key observations for the alternative reconstruction model is that the downgoing vertical component wavefield contains different crossline spatial information than the upgoing vertical component wavefield. That is, if we have a certain event with non-zero crossline incidence angle recorded in the downgoing wavefield and we trace the corresponding ray back to the free-surface, reflect it and extrapolate it back to the streamer level, we arrive, in general, at a location where the upgoing wave was not measured. However, since we are not able to do a proper plane wave decomposition (the data is aliased) we do not know how to extrapolate the upgoing wavefield back up to the free-surface, reflect it, and back down to the streamer level, without being affected by crossline alias. Nevertheless, our state of information can be described as follows:

We have samples \( V_z^{-}(t,x,y) \) and \( V_z^{+}(t,x,y) \) \( [i=1,2,\ldots,N] \) of the responses \( V_z^{-}(t,x,y) \) and \( V_z^{+}(t,x,y) \), that is, of two linear filters \( H_1(\omega,k_x,k_y) = 1 \) and \( H_2(\omega,k_x,k_y) = \exp(i2k_zy) \), resp. with input \( V_z^{-}(t,x,y) \), sampled at \( \frac{1}{2} \) the Nyquist rate in the crossline direction (and at full rate in the inline direction).

Thus, provided that we have decomposed the wavefield correctly in 3D into up- and downgoing waves and that we know the relation between the up- and downgoing wavefields, we can view the up- and downgoing waves at the actual streamer locations as the samples of the output of 2 linear systems applied to the upgoing, sampled at \( \frac{1}{2} \) the Nyquist rate.

In the above description we have used exactly the terminology used by Papoulis when describing the generalized sampling expansion (Papoulis, 1977). Thus, if we identify the linear filters and the input and sampled data as above, we can apply the theory by Papoulis to reconstruct the upgoing input without aliasing. Instead of the mixed-domain integral approach proposed by Papoulis, we use the solution by Brown (1981) or equivalents which avoid the mixed domain \((y,k_y)\) integrals and that can be applied in the crossline space or crossline wavenumber domain directly.

**Workflow**

When pressure, crossline component, and vertical component of particle velocity measurements are available, the vertical component of particle velocity can be both deghosted and crossline reconstructed (i.e., de-aliased) by applying the following workflow:

1. Crossline reconstruct the total (up+down) pressure wavefield using the crossline component of particle acceleration jointly with the pressure.
2. Filter the crossline reconstructed total (up+down) pressure wavefield with the obliquity (i.e. the vertical wavenumber divided density times angular frequency). Note that since we just need to reproduce the effect of obliquity on \( P \), we can use the unsigned vertical wavenumber obtained direct from the 3D dispersion relation.
3. Add and subtract the filtered crossline reconstructed total (up+down) pressure wavefield to and from the vertical component of particle acceleration at the streamer locations, to yield the aliased but correctly 3D decomposed up- and downgoing particle velocity wavefields.
4. Crossline reconstruct the upgoing particle velocity wavefields using the 3D decomposed up- and downgoing particle velocity wavefields, the free-surface reflection operators, and the generalized sampling expansion / multi-channel sampling theorem.

**Example**

We now illustrate the methodology using a simple 2D, plane-wave example.

In Figure 1 (a)-(c), amplitude spectra of synthetic \( P, V_y \) and \( V_z \) input data at 50m spacing are shown, respectively. A minimum phase wavelet with flat amplitude spectrum [2-65] Hz was used. The receivers are at 17.5m depth. Note the notch at 43Hz on \( P \) and \( V_y \) and at 21.5Hz on \( V_z \). The angles of incidence of the 12 events are chosen such that they alias at or above 33Hz. All velocity data is scaled with the acoustic impedance for ease of combination and display.
Step 1: In Figure 1 (d)-(f), amplitude spectra of crossline reconstructed pressure at 25m, reference pressure at 25m, and the corresponding difference are shown. The pressure was reconstructed from the P and Vy data at 50m using multi-channel interpolation. Note that the pressure wavefield has been de-aliased but that the notch, as expected, is still present.

Step 2: Next the 3D Vz decomposition operator (ckz/ω) is applied to the spectrum of the reconstructed pressure. The result at the (aliased) Vz input spacing of 50m is in Figure 1(g).

Step 3: In Figure 1 (h) and (i), amplitude spectra of the upgoing and downgoing vertical component obtained by summation and subtraction of the filtered reconstructed pressure data from the vertical velocity input data are shown. Notches have been removed, but the data are still aliased. Also, downgoing Vz has exactly the same amplitude spectrum as upgoing Vz, which implies that any crossline information is in the different phase of up- and down Vz.

Step 4: In figure 1 (j)-(l), amplitude spectra of the crossline reconstructed upgoing vertical component wavefield at 25m spacing, the reference upgoing vertical component wavefield at 25m, and the difference, are shown respectively. The 3D decomposed upgoing vertical component has been de-aliased. The small error in the reconstructed upgoing wavefield at the Nyquist crossline wavenumber can be predicted from theory (see Papoulis, 1977).

Conclusions

We have presented an alternative model for crossline reconstruction based on aliased up- and downgoing waves. The approach provides insight into the role of Vz and the ghost model in joint interpolation and deghosting and is amenable to further theoretical analysis. A simple 2D plane wave example was used to illustrate the methodology. The arguments and methods presented are not limited, however, to plane-waves or to 2D.

Acknowledgements

We thank Schlumberger/WesternGeco for permission to show these results.

References


Figure 1 Amplitude spectra. (a)-(c) Pressure, Horizontal-, and Vertical Velocity data at 50m respectively. (d)-(f) Crossline Reconstructed Pressure, Reference, and Difference at 25m respectively. (g)-(i) Filtered Crossline Reconstructed Pressure, Up- and Downgoing Vertical Velocity at 50m respectively. (j)-(l) Crossline Reconstructed Upgoing Vertical Velocity, Reference, and Difference z at 25m. See text for details.