Th N107 09

Q-compensation Through Depth Domain Inversion

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SUMMARY

We propose a new method that enables Q-compensation after depth imaging and within the framework of depth domain inversion. The approach relies on deterministic operators that capture the dip-dependent amplitude and phase anelastic effects seen by the waves as they propagate through a 3D anelastic medium, and corrects for them by inversion in the image domain. It provides fully deconvolved images (reflectivity) compensated for both anelastic and illumination effects. We demonstrate this approach in the context of reverse time migration (RTM) and show its potential with a complex synthetic.
Introduction

Over the years, more physics is being integrated into our earth models and depth imaging algorithms are incrementally adjusted to simulate more complex phenomena distorting the seismic wavelet as it propagates. Anelasticity is one of the phenomena recently tackled in such a natural, yet challenging, deterministic way.

As a seismic wave propagates through the subsurface, the anelastic nature of the earth results in dissipation and dispersion effects that can severely degrade the phase, amplitude and resolution of our depth migrated images. Continuous effort is put into trying to derive reliable 3D 1/Q models of the subsurface. Concurrently, depth imaging algorithms are extended to account for anelasticity during propagation. While the methodology and implementation are already well-established for frequency-domain algorithms or ray-based algorithms, handling anelasticity in a conventional (time domain) reverse time migration (RTM) algorithm is less straightforward. Recent research activities focus on deriving accurate, yet tractable, visco-acoustic equations that can be implemented in a RTM process in a stable and cost-effective fashion. Anelastic (time-domain) least-squares RTM schemes are also under investigation (e.g. Dutta et al., 2013). Alternatively, we propose to account for anelasticity after RTM, during depth domain inversion.

Depth domain inversion (also known as image-domain least-squares migration) uses deterministic 3D operators that capture the dip-dependent illumination and wavelet stretch effects present in the image and corrects for them by inversion in the image (depth) domain. It regards a migrated image, \( m \), as the result of a linear blurring operator, \( H \), applied to a reflectivity model, \( r \), and seeks to find the best reflectivity model by minimizing the objective function \( ||m - Hr||^2 \), using a precomputed approximation of \( H \). A general discussion about image-domain least-squares migration and how it relates to the more conventional data-domain schemes can be found in Fletcher et al., 2015.

Depth domain inversion has proven efficient in providing an estimate of the reflectivity (deconvolved image) showing more continuous and sharper events than a standard RTM image, together with more reliable amplitude information (e.g. Letki et al., 2015). However if an image has not (or only partially) been compensated for anelastic effects, a standard depth domain inversion will carry the associated amplitude and phase distortions through the reflectivity estimate. We propose here to modify the depth domain inversion scheme to account and correct for these effects.

Extending the blurring operator to account for anelasticity

In the least-squares migration framework, the blurring operator is defined as \( H = M^*M \), where \( M \) is a linear modelling (or demigration) operator linking the recorded (pre-processed) data to the reflectivity, and \( M^* \) (migration operator) is its adjoint. The Hessian operator \( H \) is a measure of illumination that reflects the effects of velocity variations and acquisition footprint and blurs the true reflectivity to give the migrated image.

Let us now consider some data, \( d_Q \), that have not (or only partially) been compensated for anelastic effects during pre-processing and an associated image, \( m_Q \), migrated with a standard (i.e. not accounting for anelasticity) migration algorithm, \( M^* \). We now regard these data as the result of an anelastic linear operator, \( M_Q \), applied to the reflectivity model:

\[
d_Q = M_Q r .
\]

The image, \( m_Q = M^*d_Q \), carries the anelastic effects and is now linked to the reflectivity through an anelastic blurring operator \( H_Q \):

\[
H_Q = M^*M_Q .
\]

Here, we are stepping away from the “least-squares” migration framework by defining a non-Hessian blurring operator \( H_Q \), for which the modelling and the migration operators are no longer adjoint with...
each other. $H_Q$ captures the additional anelastic effects undergone by the data but not accounted for in migration, and can be used to correct (de-blur) the image. Q-free reflectivities, $r$, are then derived by minimizing (using for instance a conjugate gradient solver) the least-square objective function:

$$\|m_Q - H_Qr\|^2. \quad (3)$$

We approximate our anelastic blurring operator by numerically computing its impulse response. The impulse response of the modelling and migration operators to a point scatterer is known as a point spread function (PSF). We assume here that we have a reasonable 1/Q model of the subsurface (or residual 1/Q model if the data have already been partially Q-compensated). We derive a set of anelastic PSFs by considering a 3D grid of point scatterers and running a modelling/migration exercise, using the acquisition geometry of the data and the velocity model used to migrate the original image. Similar propagation kernels are employed in modelling and migration but the kernel in the modelling exercise is extended to simulate amplitude and phase anelastic effects, using the provided 1/Q model. A full approximation of the anelastic blurring operator $H_Q$, required to solve (3), is then derived by interpolating “on-the-fly” the anelastic PSFs, during application of the convolution operator.

We apply our approach in the context of RTM and approximate our modelling and migration operators using two-way partial differential equation propagators. A visco-acoustic formulation is used in the modelling exercise, following the constant-Q standard linear solid theory, implemented through memory variables (Blanch et al, 1995). Three relaxation mechanisms are employed, which seems to enable accurate approximation of the constant-Q model over the frequency band of interest.

**Application to the BP2004Q synthetic**

We test this new inversion scheme using the 2D BP2004Q model built after (Billette and Brandsberg-Dahl, 2005) and extended to include anelasticity (Cavalca et al., 2013). Visco-acoustic data are generated using the 1/Q model shown in Figure 1a, containing two high-1/Q anomalies embedded in a non-attenuative background. The source wavelet is a Ricker wavelet centred at 19Hz. In this experiment, we assume that the provided velocity model is characteristic of the lowest frequencies. Under this assumption, dispersion translates into a phase velocity increase with frequency across the signal bandwidth. Acoustic data are also generated to benchmark our results. Both acoustic and visco-acoustic data are migrated with acoustic RTM, using the true velocity model.

![Figure 1](image_url)  
*Figure 1 (a) 1/Q model superimposed with the true reflectivity model. The dashed box indicates the area of interest, where inversion is run. (b,c) Visco-acoustic and acoustic PSFs respectively, computed in the yellow area. (d,e) Associated (normalized) wavenumber amplitude spectra for one PSF. White colour indicates amplitudes between 0 and -15dB.*
Figure 1b displays anelastic PSFs derived through visco-acoustic modelling and acoustic migration, following the approach described above, below the main 1/Q anomaly. Figure 1c displays the corresponding conventional acoustic PSFs (generated with acoustic modelling and migration). A clear dip-dependent imprint of the 1/Q heterogeneity is visible on the amplitude and phase of the anelastic PSFs. Dips illuminated by waves propagated through the 1/Q heterogeneity are strongly attenuated, especially at high wavenumbers, as illustrated with the wavenumber amplitude spectra displayed in Figures 1d and 1e.

We use the acoustic PSFs and run conventional depth domain inversion with the RTM images derived using the visco-acoustic data and the acoustic data (benchmark). Inverted reflectivities in the area of interest are displayed in Figures 2a and 2b, respectively. Anelastic effects present in the RTM image derived using the visco-acoustic data are carried through the associated reflectivities which suffer from dimmed amplitude, together with distorted and shifted events in the two areas below the 1/Q heterogeneities. Figure 2c shows the reflectivities obtained by running depth domain inversion using the anelastic PSFs. Dip-dependent 1/Q effects are effectively captured in the PSFs and corrected during inversion, leading to reflectivities quite similar to the ones derived with the acoustic data, and close to the true reflectivities (Figure 2d).

**Figure 2** Reflectivities derived through conventional depth domain inversion using the visco-acoustic (a) and acoustic (b) data. (c) Reflectivities derived with our anelastic depth domain inversion approach. (d) True reflectivities. Areas of strong attenuation are highlighted with the dashed ovals.

**Discussion**

The visco-acoustic propagator employed in this paper couples amplitude and phase anelastic effects. Whilst we can apply residual Q-compensation by defining a residual 1/Q model, we cannot separate amplitude and phase effects. Alternative propagators that rely on decoupled dissipation and dispersion operators (e.g. Zhu and Harris, 2014) could be used in the proposed depth domain inversion approach to make the whole processing workflow more flexible and enable independent phase or amplitude compensation during inversion.

As in any Q-compensation exercise, noise amplification may be an issue. Care has to be taken during inversion to avoid fitting the noise present in the image and not captured in the PSFs, especially at...
high wavenumbers for which the attenuated signal can be significantly weakened. The number of iterations is a critical parameter and spatially-variant weights on the image or on the model (i.e. on the reflectivities) can help control the inversion in areas where the S/N limit is reached. Running the inversion in the depth domain has the advantage of enabling a relatively easy definition and tuning of constraints on the model. The least-squares optimization problem in (3) is, in practice, extended to accommodate regularization and a priori constraints. While in the previous synthetic example, no constraint was used, with real data, we observe that applying for instance some level of structural smoothing during inversion helps in mitigating noise amplification.

In this paper, the inversion is run post-stack. While the 3D 1/Q effects occurring as the waves propagate are effectively captured in the (stacked) anelastic PSFs, we could think of running the inversion pre-stack, to access pre-stack Q-compensated reflectivities, required for earth model attribute pre-stack inversion.

**Conclusion**

We have introduced anelastic blurring operators to correct for amplitude and phase anelastic effects, post-imaging, during depth domain inversion. The approach is applied in the context of reverse time migration by approximating the anelastic blurring operator through the computation of visco-acoustic point spread functions. Visco-acoustic PSFs effectively capture both the illumination and anelastic effects, induced by the acquisition geometry of the data, the velocity complexity and the 1/Q distribution of the earth. They can be used to de-blur the image and derive, if the velocity and 1/Q models are reliable, reflectivities fully corrected for both effects. The approach gave promising results on synthetics and real data (not shown here). Note that, if necessary, a Q-compensated image (compensated for anelasticity but not for illumination effects) can also be computed by re-convolving the inverted reflectivities with conventional acoustic PSFs.

**Acknowledgements**

The authors thank BP and Frederic Billette for creating and providing the original 2004 BP model and Schlumberger for permission to publish this work. Many thanks also to Zhen Xu and Michel Vie for developing the software used in this study.

**References**


