New Dimensions in Modeling Resistivity

Over sixty years ago, resistivity modeling emerged as a means to design electrical logging tools and interpret their responses in simple, layered formations. Computational advances now make it possible to rapidly predict unexpected responses from modern tools in complex formations, and to more accurately extract true resistivities and structural geometries from resistivity logs.
In 1927, Conrad Schlumberger changed the course of petroleum exploration when he purposefully tipped his surface electrical prospecting array vertically and sent it logging down a wellbore. Existing resistivity modeling methods were applied to the new geometry to predict the vertical array response. Thus modeling is not new; but it is finding new uses.

In the logging vernacular, the term modeling refers to computing a logging instrument response in the presence of the environment surrounding the logging instrument. The model attempts to capture all the detail required to account for and duplicate an observed instrument response, and it is used as an aid to log interpretation or tool design.

A resistivity model consists of a resistivity distribution for the environment and a description of the instrument sensors. The resistivity distribution includes the resistivity of various borehole and formation regions and locations of their boundaries. Depending on the logging situation, the boundaries might include the locations of geological bedding planes, borehole wall, invasion fronts, and the relative angle between all of these and the instrument axis. The result of modeling is a synthetic tool response, or log, which can be compared to an observed log for interpretation or to the resistivity values in the model for tool design (see "What is a Model?" page 43).

In the early days of resistivity modeling, scale models and analog computers—for example resistor networks—helped to characterize tool responses in cases too complicated for mathematical models, such as when radial and vertical resistivity variations were to be considered simultaneously (above). The resistor network became one of the earliest applications of resistivity models in the creation of log correction charts describing the effects of one-dimensional (1D) and two-dimensional (2D) environmental heterogeneities (such as changes in the vertical direction like shoulder beds and in the radial direction as with invasion) on logs. Typical cases were computed for the charts, but intermediate or extreme cases encountered in practice required interpolation within a chart or among several charts. The drawback to this kind of modeling was the difficulty in varying the parameters of the problem; each new case required a new scale model to be built or hundreds to thousands of resistors to be exchanged.

With the development of digital computers, direct numerical solutions of the electromagnetic equations can be used to compute resistivity response in models having complex geometries. To achieve this, these equations are approximated by a large system of simultaneous equations; the next task is to find numbers that simultaneously satisfy all the resulting equations. The geometrical restrictions are much less severe than analytical and scale models. Realistic problems can be formulated, and the solutions appear as simple and unthreatening numbers rather than as arcane and difficult-to-compute mathematical functions. Unfortunately, the computational burden is enormous; solutions for realistic three-dimensional (3D) problems by this method require extensive machine computation (see "The Vocabulary of Resistivity Modeling," page 46). On the other hand, machine capability has steadily improved for more than 50 years, and it is now possible to obtain useful numerical solutions to formerly intractable problems.

Most log analysts appreciate that tool-response modeling plays an important role in instrument design. Less well recognized is that both the apparent resistivity recorded by the logging tool itself, and the various corrections thereto, are derived from mathematical models. The models help relate tool responses to true formation resistivity, $R_t$.  

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In the last decade, modeling provided the basis for automatic environmental correction algorithms such as Phasor processing, and is the basis for the log products of the AIT Array Induction Imager Tool. For both of these tools, shoulder effect and thin-bed response are corrected to the resolution limits of the tool at low dip angles. Further model-based processing allows response corrections for moderate dip angles (< 50°) for these tools.

Although modern resistivity tools perform most environmental corrections as the log is being recorded, oil company archives are bulging with logs run with older DIL Dual Induction Resistivity Log and SFL Spherically Focused Resistivity tools and equivalents from many service companies. For these tools, modeling is the only way to understand the complex effects that caused their logs to read completely wrong in many situations.

Tool-response modeling, as practiced in modern log interpretation, extends the interpretation of instrument responses beyond the simple resistivity distributions envisaged in the apparent resistivity function and its first-order corrections—as provided in chartbooks—to more realistic geometrical arrangement of the borehole and formation environment. The idea is to discover the probable distribution of resistivity that would give rise to an observed tool response. In many cases, only tool-response modeling can decode what the instrument responses reveal about the formation being logged—in many cases, if not most, intuitions gained from study of, and experience with using, charts are misleading, because the charts are not applicable.

Resistivity modeling was reviewed in Oilfield Review five years ago. The two major advances chronicled at the time were geosteering with logging-while-drilling (LWD) resistivity tools and the combination of analytical and numerical modeling methods to speed calculations. At that time, two needs were identified. First, modelers required enhanced computational capability to develop models in the 3D domain for more realistic analysis in complex formation geometries. Such improvements—already on the horizon—would, in part, come from developing new advanced algorithms that speed up lengthy calculations. Second, further technological improvements were needed to help log analysts model logs quickly enough to be practical. Some of this capability was already under development in some oil company research groups.

This article discusses the progress made in the past five years. We first show how 3D modeling is used for strongly dipping formations and horizontal well evaluation. Then, we discuss the impact of recent computational breakthroughs in modeling capabilities and practical interpretation using resistivity modeling.

Why Model Resistivity? Reserves estimates are based on log-derived measurements of formation resistivity. All other factors being constant, as the formation hydrocarbon content increases, so does the formation resistivity. In fact, using $R_w$, Archie's law allows computation of the fractional volume of the formation containing hydrocarbons. However, a few operators use raw resistivity logs as a hydrocarbon indicator—where resistivity is high, they perforate the well. This is not the best use of resistivity data.

The log analyst looking at resistivity logs from older tools—uncorrected for environmental effects—often has little to serve as a guide for resistivity interpretation. Many times the uncorrected resistivity, for example from the deep induction tool, is considered an accurate enough estimator of $R_w$. This may sometimes be true, but all resistivity instrument responses, whether from induction or laterolog tools, are influenced by the resistivity distribution in a large volume surrounding the logging instrument. Correct interpretation would require many corrections for bed boundaries, borehole, invasion and other environmental or geometrical effects.

This means that what looks like the best, or highest, resistivity reading on a raw induction log may simply be an artifact of some nearby boundary layer of contrasting resistivity. Many high-resistivity anomalies do not correspond to resistive beds at all, but rather to the interface between two beds, each with lower $R_w$ than the apparent resistivity, $R_{app}$, indicated on the log. Under simple enough conditions, such as in a vertical well through thick beds with simple invasion, chartbook corrections can suffice. For these applications, many software suppliers offer chartbook-based corrections for borehole and invasion for the DIL tool.

Wells are not always ideal for logging. Low-cost drilling efforts frequently lead to boreholes that are not uniform. Deep invasion also affects tool responses. In the last decade, thousands of deviated and horizontal wells have been drilled to optimize productivity. These wells have logging tool-toformation orientations not contemplated in the design of the apparent resistivity response and the corresponding chartbook corrections. Resistivity responses in such formations appear counterintuitive, even weird, and are not correctable by chartbook algorithms.

However, the physical principles embodied in Maxwell's equations—of such instrument responses are well understood, and with modeling capable of honoring enough detail of the tool and environment, such responses are predictable. Artifacts such as polarization horns, which appear as large transient overshoots in the tool resistivity response at bed boundaries, can be understood on the basis of such models.

Another Dimension in Resistivity Modeling

The growing interest in complex formation geometries, such as dipping beds, invasion and anisotropy, has led to progress in development of sophisticated models and enhanced computational efficiency. As interpretation problems have become more detailed, model requirements have grown from 1D, to 2D, and finally to 3D codes, to handle the geometry of the problems at hand. Significant recent developments have been the use of new numerical techniques, which make solutions in 3D models much faster. Now, they are beginning to be used for difficult log interpretation problems.

Compared to current methods, early modeling codes were slow in execution and therefore cumbersome to use for forward modeling. Models had to be modified at each iteration by editing detailed text files, and comparison of observed and synthetic responses required hard-copy plotting of the synthetic log, and visually inspecting for differences at each iteration. Nevertheless, the value added was significant.

Predicting the response of a logging tool is a complicated, detailed process. The boundaries of regions of differing resistivity (or other material electromagnetic properties) and the properties within each region must be specified by the model. The tool must also be introduced into the model with its transmitters and receivers located at specified points, with the transmitters exciting the model medium in some specified manner. The interaction of the disturbance produced by the transmitters proceeds through the medium and is detected by the tool’s receiver coils or electrodes.

The response of the each receiver is thus predicted. Resistivity modeling belongs to the class of so-called boundary-value problems. In relatively simple cases, these problems may yield analytic formulas, but, in general, a system of equations must be solved explicitly (though approximately) for each new case considered—such as when the tool is moved or the model changed—and the result is a set of numbers rather than a formula. This type of mathematical modeling is called numerical modeling. Two examples of numerical modeling in use today are the finite-element method (FEM) and finite-difference method (FDM).

Given a system of equations governing a tool response, one simply plugs in model parameters and after computation, the desired tool response quantities are written down. The result is a ray of tool responses that would be observed if a physical experiment were performed. This is called the forward problem. For cases where the governing equations are linear in the resistivities, this process is partially reversible—given the tool response, the model parameters can be estimated by multiplying the vector of observed tool responses by the generalized inverse of the same matrix used in the corresponding forward problem. This is called the inverse problem.

There are variations upon this theme. For some systems, the output of the system—tool response—can be represented as the mathematical operation of convolution of the tool input with the impulse response of the tool. Given the output (a log), the input (a formation) can be correctly determined if the transfer function (the Fourier transform of the impulse response) does not pass through zero for any value of its argument. Under this restriction, the input is determined from the output by the process known as deconvolution.

Unfortunately, this is not the case for many tools. For example, the 6FF40 is known to have a blind frequency corresponding to about 5-ft intervals. As a result, there is a nonunique inversion—for practical purposes, the thin-bed response at blind frequencies turns out to be indistinguishable from the whole-space response.

Moreover, the response of resistivity tools is usually nonlinear with respect to formation resistivity because of skin effects. Thus, in the strict sense mentioned above, neither deconvolution nor inversion is possible—at least not in a single, deterministic step. Furthermore, the forward problem itself is relatively difficult to solve. It typically is represented by a large system of equations. The solution of the equations may be relatively straightforward, but since the system is large, extensive computational resources and time are required.

The inversion or deconvolution of a nonlinear system requires that the forward problem be “linearized” by some means. Phasor processor of an induction log response provides an example of linearizing an instrument response. The induction tool’s receiver in-phase or resistive voltage, the R-signal, is deconvolved using a filter designed to correct the zero resistivity response. The tool’s quadrature or reactive voltage, the X-signal, is processed with filters and fitting functions to approximate the difference between the actual R-signal and a linear estimate of the R-signal. The two are added, and the result is a linear output with the shoulder effect corrected. In enhanced resolution Phasor processing, the log resolution is corrected for beds as thin as 3 ft [1 m].

A more versatile technique finds a solution by iteratively solving the forward problem. If the iterations are performed manually, the process is called forward modeling. If the iterations are carried out by an algorithm seeking to minimize the differences between the observed log and a synthetic log calculated in a model by adjusting the model to reduce the differences at each step, the process is called inverse modeling. Note that for either technique, an important part of the process is an efficient means of solving the forward problem so that the differences can be obtained rapidly and the iteration carried forward as fast as possible.

Equally important, especially for inverse modeling, is the construction of the initial model. If the initial model is good enough, the differences between observed and synthetic logs vanish on the first comparison. Failing this, the closer the initial model is to the actual formation, the fewer the iterations required, and the final solution is reached rapidly. Thus, the best possible initial model is essential to inversion. An initial model would be based upon the highest resolution data available plus any available estimates—such as

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using FMI Fullbore Formation MicroImager readings to get bed boundaries—of radial resistivity variation within the bed boundaries.

The type of inverse modeling described above is known as model-based inversion. This means that the form that the solution is most likely to take is decided in advance, and only solutions of that form will be found by the technique.

Differences Between FEM and FDM
In the finite-element method, Maxwell's equations are written as integral equations of the resistivity and electromagnetic fields. The next step is a process of discretization throughout the medium surrounding the tool and formation—the step of converting continuous equations to a finite dimensional system of equations that can be solved with a digital computer. For FEM, discretization is done based on a variational principle. For example, this could mean the total energy of the system is minimized at points on a gridded network throughout the media environment (above right). This is a reasonable way to solve the problem, since the static electromagnetic potentials always adjust themselves to seek a minimum energy. The topology of the discretization is usually selected to conform with the problem geometry. The discretization step leads to a large set of simultaneous equations for the electromagnetic field in terms of the media resistivities, which are represented by a set of large matrices.

The flexibility of FEM translates to complexity in numerical implementation and the codes can be more difficult to optimize. The form of the matrices is unstructured, having nonzero elements spread throughout the matrix, because these matrices reflect the topology of the discretization. The final step is to find a solution to the matrix equations representing the electromagnetic fields. For a problem of a given size, the FEM method is capable of the highest conformity to geometrical complexity of any numerical method, but is computationally slow.

The finite-difference method uses discretization based on a direct difference approximation of the differential operator form of Maxwell's equations. This leads to a grid requirement restricted to a Cartesian topology (right). Although the grid topology does not conform to the geometry of the resistivity discontinuities found in the formation environment, it can be made to approximate this geometry. The real benefit of this approach lies in the next step, in which the matrix equations for the discretization of the differential equations are usually structured because of the Cartesian topology, and always sparse because the derivatives are local operators. With these advantages, matrices lend themselves to fast, specialized computational methods that allow rapid solutions to resistivity models with extremely complex geometry.

3. Lovell, reference 12, main text.
Consumers of well log data concluded that to gain maximum benefit from resistivity logs, the numerical modeling of tool responses would be the standard for the 1990s. Several oil companies, among them Shell, BP, Chevron and ARCO, began to develop or acquire in-house modeling capabilities. And one major oil company, Mobil, formed a task force of researchers, practicing petrophysicists, computer scientists and academic consultants to make logging tool response modeling convenient, practical and routinely applicable. This commitment, combined with several technical breakthroughs, brought rapid success to this effort (see “An Efficient User Interface for Resistivity Modeling,” page 49).

As early as the late 1980s, computational efficiency was benefiting from the newly developed fast Hankel transform (FHT) applied to induction log modeling, when Richard Hardman invented a method of optimizing the use of the FHT. The number of computations was further reduced by exploiting various symmetries of the induction antenna array and reciprocity. Available by 1991, the resulting 1D computer programs were from 100 to 600 times faster, depending on dip, than the standard codes used by University of Houston Well Logging Laboratory. A recent revision to Hardman’s 1D induction modeling will be available in Geolog software from MINCOM, Pty. It uses shallow measurements, such as SFL logs, as input to assign bed boundaries. Model viewing and editing are performed rapidly through a graphical interface.

During the last decade, at the Central Geophysical Expedition in Moscow, Vladimir Druskin and Leonid Knizhnerman—using 3D modeling techniques originally developed for surface electromagnetic prospecting problems—developed a breakthrough in a new, efficient 3D finite-difference method (FDM) for resistivity modeling. Their approach, along with other recent developments in their mathematical techniques, has led to remarkable improvements in the speed and accuracy of full 3D resistivity modeling capability for both induction and 2-MHz propagation tools.

Horizontal and Highly Deviated Wells—3D Resistivity Modeling
Interpretation of resistivity tools in horizontal or highly deviated wells is far more complicated than in vertical wells, particularly when drilling-induced resistivity changes are important to the tool response. Resistivity tools have been designed for focusing in vertical wells, where horizontal layers and axial symmetry are assumed. When these conditions hold, interpretation procedures are straightforward—the recommended procedure is to apply borehole corrections, process to improve or match vertical resolution, and correct for invasion, in sequence. Although dip-induced shoulder effects can be corrected for angles up to 60°, in practice invasion complicates the interpretation. At higher angles, shoulder effects can be different on all sides of the sonde, and invasion may be highly nonsymmetrical. As a result, apparent resistivity response interpretation using the standard methods is not possible in horizontal and highly deviated wells.

For laterolog-type tools, such as the DLL Dual Laterolog Resistivity tool and the RAB Resistivity-at-the-Bit tool, effects of the borehole and possible invasion cannot be ignored. In the presence of dip, such tools must be modeled with 3D codes. Fortunately, their low-frequency operation means that they can be modeled with Poisson’s or Laplace’s equation—a significant simplification of Maxwell’s equations—and finite-element method (FEM) codes for these equations are relatively fast to compute. For example, a full 3D FEM solution for the DLL response in the presence of dip, pinchouts, anisotropy, invasion and tool eccentricity takes 10 to 15 seconds per tool position.

For induction-type tools, such as the AIT response and CD R Compensated Dual Resistivity response, the effects of resistive mud drilling-induced alterations and borehole signals can be negligible, but the frequency effects on the tools are not. Thus, the first attempts to model their response in highly deviated wells were based on solving for the electromagnetic fields in layered 1D geometries, which omitted both borehole and invasion. These early attempts have met with reasonable success. This class of models is widely applicable, fast to compute, and commercially available. Subsequent efforts based on 2D models have included a borehole and axisymmetric invasion; these are also commercially available and fast enough to be practical for constructing model-based interpretations. Unfortunately, nonaxisymmetric invasion also exists, even at early stages of invasion encountered by LWD tools. Even when invasion is axisymmetric, in the presence of dip this geometry requires a 3D solution.

Fast FDM modeling techniques are now available for solving the full 3D electromagnetic problem necessary for interpreting induction logs in deviated wells with invasion, and for understanding tool responses in other difficult environments including complex fracture systems, faults and pinchouts, and effects of resistivity anisotropy. Finally, realistic logging problems, with combined effects of invasion, dip and shoulder beds, can be analyzed within reasonable time using the new 3D FDM induction forward-modeling programs.

The Vocabulary of Resistivity Modeling

Geometrical Dimensions in Modeling
The physics of resistivity modeling is described by Maxwell’s equations. Depending on the assumptions made about the spatial distribution of resistivity, these solutions can take the form of simple formulas, giving the electromagnetic field for any desired point in space. In complicated cases, the field cannot be expressed in terms of simple formulas, but must be laboriously computed at many points in space simultaneously. The degree of complexity is customarily summarized by referring to geometries of differing dimensionality—zero-dimensional (0D), one-dimensional (1D), two-dimensional (2D) and three-dimensional (3D).

One way to decode this jargon is to consider how resistivity varies in a coordinate system. Consider, for example, the cylindrical coordinate system specified by the coordinates \( \rho, \phi \) and \( z \). If the resistivity, \( R \), is constant everywhere, then it is referred to as 0D. If the resistivity, \( R \), in a geometry varies with only a single coordinate direction, then that geometry is referred to as 1D. The familiar layer-cake geometry of sedimentary geological structure is an example of how the resistivity varies only in the \( z \)-coordinate direction or, \( R = R(z) \) (below left). In essence, each layer has a constant resistivity, but the resistivity varies between the layers.

Another commonly used geometrical arrangement in well logging features a cylindrical wellbore surrounded by coaxial cylindrical shells bounding cylindrical regions of differing resistivity (left). In this case, \( R = R(\rho) \). Thus, 1D geometries are either vertically or radially layered geometries.

Similarly, if resistivity varies in the vertical and radial directions and is axisymmetric, then the geometry is two-dimensional (below right). In this case, \( R = R(\rho, z) \) in the notation introduced above.

When \( R = R(\rho, \phi, z) \), the geometry is said to be three-dimensional (next page, top). This system of nomenclature can be extended without modification to cases in which resistivity at a point varies with direction—resistivity is a tensor.

To illustrate this, Maxwell’s equations are formulated for the electric field of a magnetic dipole in an infinite and homogeneous conductive medium, and the result is a simple formula. The medium is zero-dimensional, and the formula is the basis for calibrating the induction tool’s voltage measurement to apparent resistivity. If the equations are formulated for the electric field of a vertical magnetic dipole on the axis of a radially layered medium or within a horizontally layered medium, the results are still formulas for a speci-
The field can be computed without reference to the field at other points, but the formulas are complicated and contain semi-infinite integrals to be performed, after substituting parameter values, to get a numerical answer. The geometries are one-dimensional; these computations form the basis for invasion corrections (radially layered 1D geometry) and bed-thickness corrections (vertically layered 1D geometry), and most commercially available 1D codes.

Models with geometries of higher dimensionality usually cannot be solved analytically—the solutions cannot be expressed as simple formulas. In these higher dimensional cases, numerical methods must be used to obtain the field values. Although there are many different numerical methods, they all share the same property: to compute the field at a single point, the field must be known at many adjacent points. For some methods, the field must be computed at every point, and these computations are time-consuming. Consequently, computations in 2D are about three times slower than 1D computations, and 3D geometries require two orders of magnitude more time than 1D computations.

A possible source of confusion arises when more realistic modeling is attempted. To illustrate, consider a vertically oriented, point magnetic dipole on the axis of a borehole. The induced electric field is concentric and symmetrical with the axis of the borehole and is therefore independent of the azimuthal angle Φ.

However, if the dipole is moved off the axis of symmetry to represent an eccentered tool in a borehole, the cylindrical symmetry of the electric field is broken; now two components of the electric field are required—radial and azimuthal—to completely describe the electric field.

Numerical modelers sometimes refer to the dimensionality of their models according to the number of non-zero field components necessary to completely specify the field. However, in this example, the medium remains 2D irrespective of the source location or orientation, even though the mathematical formulation may have to account for a greater or fewer number of field components according to the source orientation and position.

1. This geophysics convention is adopted in this article.
2. The third possibility for a 1D geometry, \( R = R(\Phi) \), does not correspond to a case of much utility for resistivity modeling of induction tools, which have no sensitivity to \( \Phi \) variations, and is not used.
Three-dimensional model for an invaded, dipping reservoir. The overlying and underly-
ing formations are shales, while the reservoir, containing 90% oil saturation, is invaded to
15 in. [38 cm] by saline water.

AIT Array Induction Tool logs in dipping
bed with invasion. The results of the 3D
modeled induction responses (blue curves)
show the 10-in. response in agreement
with the invaded zone resistivity, \( R_{oi} \)
(dashed green line). However, in the cen-
ter of the thick hydrocarbon zone, the
deeper tool responses are reading only
about 40% of the true formation resistivity,
\( R_t \) (solid green line). This corresponds to
an oil saturation of only 60%, a significant
error. The deepest reading 90-in. response
also shows strong polarization horns at the
center boundaries.

The effect of invasion without dip. In the
absence of dip, the case of a borehole
through an invaded layer can be com-
pletely modeled in 2D (red curves). Treat-
ing the problem in 3D yields the same
results (blue curves). Note that the deep
90-in. tool response is greater than the \( R_t \) in
the center of the bed, while the shallower
60-in. tool response is nearly equal to \( R_t \).
Also note the absence of horns at the bed
boundaries. These effects are different
from those seen in dipping formations.

The effect of dip without invasion. In the
absence of invasion, the case of a borehole
through a dipping layer can be com-
pletely modeled in 1D (red curves). Treat-
ing the problem in 3D should, and does,
yield the same results (blue curves). In this
comparison, the bed boundary horns
appear on the shallow 10-in. reading as
well as on the deep 90-in. tool response.
The deep 90- and 60-in. tool responses are
reading much less than \( R_t \) while the shal-
low 10- and 20-in. responses are close to \( R_t \)
in the center of the thick bed.
Modeling the Effects of Dip and Invasion. An example comes from a case in which the AIT tool logged through a 20-ft [6.1-m] resistive bed at an angle of 60° from vertical (previous page, top). The shallow 10-in. log reads near flushed zone resistivities, $R_{\text{xo}}$, as expected. However, the deep 90-in. curve falls way below $R_x$, and the 20-, 30- and 60-in. curves cross each other in a disorderly fashion, indicating a mixture of shoulder bed and invasion effects (previous page, bottom left).

To disentangle the different effects and compare the relative effects of invasion and shoulder beds, the limiting cases of invasion with no dip (previous page, bottom center), and dip with no invasion were modeled (previous page, bottom right).

First, in the case of invasion with no dip, the expected curve separation occurs, with the 10-in. curve reading close to $R_{\text{xo}}$ and the deeper reading curves increasing in sequence towards $R_x$ with the 90-in. curve reading above $R_x$. Next, dip with no invasion results show that there is a significantly large shoulder-bed effect on the deep curves in this 20-ft bed at 60° dip. At center-bed, the readings decrease in sequence from the shallow to deep curves, with the 90-in. center-bed curve reading only 25 ohm-m in the 100-ohm-m bed.

Comparing the combined dip and invasion results with the results of the nondipping calculations shows the 10-in. curves read the same in the nondipping bed. The 20- and 30-in. dipping curves read a bit lower than those in the nondipping bed, indicating a slight shoulder effect. Comparing the combined dip and invasion results with those of the noninvaded dipping bed model shows that the 60- and 90-in. curves behave the same, in both cases reading below $R_x$. This indicates that the deep-reading logs are not influenced by the shallow invasion, but that there is a considerable shoulder-bed effect, which in this case of conductive invasion ($R_{\text{xo}}$ less than $R_x$) could be mistaken for deeper invasion. Modeling of dip and invasion together and separately helps to interpret this difficult case. It is important to archive all AIT raw tool logging responses in order to take full advantage of the new, faster 3D modeling capability.

Before computational codes were improved, the time the log analyst spent editing the model geometry file between computations was comparable to computational time. By 1991, 1D induction responses could be computed at the rates of hundreds of feet per second. As a result, editing time subsequently came to be viewed as a major bottleneck. The Mobil Resistivity Modeling Task Force was formed to specify and develop a graphical user interface capable of displaying observed log curves in a user-selectable format, and also capable of rapidly creating, deleting and otherwise modifying resistivity models in the same windows as the data using the point, click and drag features available in the then new X-windows system.

The interface is not itself a numerical modeling code; rather, it is a graphical editor useful for quickly constructing and modifying models. It incorporates facilities to rapidly and automatically build initial models using user-specified log response curves, and is useful for the specification of 2D axisymmetric models and 1D models with constant relative dip. Commercially available modeling codes currently adapted for use include forward and inverse 1D for 6FF40 and ILD deep induction logging, 2D for conventional electric survey, 2D for laterolog and spherically focused logs, and 2D for induction. Other suitable codes are under development by various vendors. The Oklahoma benchmark formation can be computed in well under one second; log responses in 2D geometries take significantly longer (though not prohibitive) times to compute over similar intervals. However, the incentive (and reward) for 2D modeling is the same as for 1D modeling—hydrocarbon pore volume increases on the order of 5 to 15% from a more accurate $R_t$ analysis.

The resulting program was being routinely used by Mobil log petrophysicists by 1992. The marriage of fast, 1D induction forward modeling to the graphical user interface made for ward modeling a practical tool for routine log interpretation. Mobil licensed its graphical user interface to Z&S Consultants, Inc., where it has been available under the trade name R-BAN since about 1993; to date it has been licensed to a number of major and independent oil companies. Petrophysicists experienced with the program and familiar with the 6FF40 response regularly model 3000 to 5000 ft [900 to 1500 m] per day.

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16. As a test, to verify that the curve crossover was not simply an artifact of 3D code inaccuracy, the FDM code was compared with an analytical solution in both cases, since each limiting case can be solved in a 2D analytical model. Agreement within 3% was achieved in all cases.
Shallow (24-in. diameter) invasion in a horizontal well approaching an overlying shale. Here the deep-reading 3D modeled 60- and 90-in. curves (blue) both track the no-invasion 1D results (red) and approach R_x as the tool moves farther away from the boundary, while the shallower 10-, 20-, and 30-in. curves are all shifted towards R_xo. In all cases, far from the bed boundary, the FDM results approach those of the no-bed-boundary, invasion-only 2D model (black points).

Deep (48-in. diameter) invasion in a horizontal well approaching an overlying shale. In this deep-invasion geometry, only the deepest 3D modeled 90-in. curve (blue) tracks the no-invasion 1D result (red). The 10-in. curve reads R_xo, and the 20-, 30-, and 60-in. intermediate curves are shifted toward R_xo. The 3D FDM calculation is carried up to, but not through, the bed boundary because of uncertainty about the shape of the invasion just below the boundary. In all cases, far from the bed boundary, the FDM results approach those of the no-bed-boundary, invasion-only 2D model (black points).

Anisotropic invasion profile. The effect of invasion at 1-week intervals is shown for six borehole deviations from vertical to horizontal in a formation with a permeability anisotropy k_horizontal/k_vertical of ten to one. The invasion profile is characterized by its aspect (long axis to short axis) ratio, which depends on permeability anisotropy, time and dip.
Invaded Horizontal Well Near a Cap Shale. Horizontal well logs in any environment are difficult to analyze without 3D modeling. A typical and interesting problem is the case of invasion in a permeable sand below a cap shale interface (previous page, top left and center). In these examples, the logging sonde remains parallel to the bed boundary, while the distance between the drainhole and the boundary is varied.

The 3D FDM model is computed for shallow (24-in. diameter) and deep (48-in. diameter) invasion depths and compared with the results of a 1D analytical model without invasion. The 1D model shows the typical curve order reversal and polarization-induced horns on the induction responses at the bed boundary. Also, for comparison, the no-bed-boundary, invasion-only limiting responses were computed with a numerical 2D model.

These results show that for shallow to moderate invasion, the deepest curve can be used to infer R_t and proximity to the shale cap, while the shallowest curve indicates R_x. The relative separation between the intermediate curves can be used with caution to estimate depth of invasion. It is clearly still possible to get good results from the AIT induction in horizontal wells, even though the tool was designed for vertical wells with axially symmetric geometries.

Asymmetric Invasion. As a refinement on the above model geometry, which assumes a cylindrical invasion front, it is interesting to examine the causes of nonuniform invasion, and then forward model the tool response in such environments. Nonuniform effects can be caused by gravity segregation or permeability anisotropy. As a well deviates from vertical to horizontal, gravity can cause invading fluid to behave differently in the top and bottom halves of the borehole. Permeability anisotropy causes an elliptical invasion shape in deviated wells. The invasion shape becomes more elongated relative to the borehole as permeability anisotropy, well deviation and quantity of filtrate invasion increase (previous page, right).

The 3D FDM computes the effects of the shape of the invasion profile, after three weeks of elliptical invasion, on AIT responses (left). The results show that volumetric contribution of R_t is increasing as the shape of the equal area invasion front becomes more elongated. The sensitivity to R_t becomes greater because a larger portion of the induction current circulates outside the invaded zone as the front elongates. The modeled AIT responses directly reflect the relatively shallow invasion depth in the vertical and horizontal portions of the well. The filtrate invades preferentially in the horizontal direction along the higher k_horizontal and less filtrate invades in the well’s vertical direction. In the horizontal borehole, the formation fluid remains closer to the logging tool above and below the drainhole, which increases the influence of R_t on the shallow resistivity curves.

17. A cap shale is frequently called a shale-seal or simply a seal, because it represents an impermeable barrier on top of a reservoir.
Gravity segregation of invasion in a horizontal well. Gravity sweeps the denser water downward (left), reducing the invasion on the top side while producing a hydrodynamically unstable mixing of the invading filtrate with movable oil on the bottom side of the drainhole. Geometry for gravity segregation in a horizontal well is modeled as a box (right).

Field Logs and 3D Modeling. Computer modeling predicts many 3D effects, but what do real logs look like? A field log from the Middle East in a horizontal well with moderately salty mud invasion illustrates the need for 3D modeling in interpreting induction logs in deviated wells (next page, top right). Below 150 ft, the logs reveal signs of the horizontal wellbore approaching a shale bed from below—the deep 90-in. curve with signs of horns and the 20- and 30-in. curves in reverse order read lower than the shallow 10-in. AIT and the MicroSFL logs. However, the low-reading 20-in. curve could not be modeled by a single interface with known shale resistivity of 4 ohm-m.

Buoyancy, caused by gravity, is another phenomenon that can greatly distort the shape of the invasion front (top). The water filtrate invading an oil zone in a horizontal well behaves much differently on the bottom side than on the top side of the drainhole. As a result, the resistivities of the invaded zones at the top and bottom of the drainhole are quite different. On the top, $R_{xo}$ is confined to a rather shallow invaded zone, but below the resistivity is a gradual transition—from $R_{xo}$ to $R_s$—over a relatively thick region. Modeling the gravity-induced asymmetric invasion profile in a horizontal well shows that the effect on AIT responses is noticeable only on the most shallow curve—the 10-in. response (above).

The low-reading 20-in. curve along with the reversed curve order suggests an annulus invasion profile. A short zone from 100 to 250 ft was chosen for modeling. With porosity log data, a known formation water resistivity, $R_{sw}$, and the tool response, a formation model with an annulus profile was generated. From the earlier discussion on cap shales and invasion, it is reasonable to assume that the shallow AIT logs (10-, 20- and 30-in. curves) respond to the invasion profile, while the deeper logs (60- and 90-in. curves) respond to nearby bed boundaries.

In the formation model, invasion occurs in the sands with only slight variations in the depth of invasion to fit with variations in porosity and deep induction response (next page, left). The effect of the shale was to cut off the upper part of the invasion. Thus as the invasion and annulus profile approached the cap shale, they are truncated at the bed boundary (next page, bottom right). As modeling progressed, it became clear that the behavior of the 90-in. log is influenced as much by invasion as it is by the bed boundaries. Only when the invasion radius was adjusted appropriately were the excursions to 2000 ohm-m reproduced.

The qualitative agreement between the field logs and modeled logs suggests that the annulus profile is real. The behavior of all the logs is a complicated mixture of invasion effects and high-angle, bed-boundary effects. This model ignored any gravity or permeability anisotropy effects, because these are negligible compared to the first-order effects of bed boundaries and invasion, and because the sensitivity of the AIT logs to these effects is not large.

All the unknowns surrounding invasion in horizontal wells lead to the non-uniqueness of any solution that matches the field logs. Additional information, such as $R_{xo}$ or shale resistivity, $R_{shale}$ becomes essential to help limit the uncertainty. This example shows that, just as in vertical wells, it is inadequate to assume a simple invasion model in horizontal wells. Only modern multiarray tools and full 3D modeling can successfully provide quantitative interpretation of resistivity logs in horizontal wells.

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18. The invasion depths are strongly influenced by the formation fluid mobility (permeability/viscosity): high mobility results in shallow top and deep bottom invasion depths. See Anderson et al., 1996, reference 15.
Field logs from a horizontal well underlying a cap shale. The GR response increases from 300 ft to 150 ft as the borehole approaches the cap shale.

3D formation model with annulus profile geometry in a horizontal well. $R_t$ was 200 ohm-m. The borehole was modeled at the shale-formation interface (top) and at 6 in. (middle) and 12 in. (bottom) below the interface. No invasion was permitted into the impermeable shale.

3D simulation results of AIT logs (top) in a horizontal well with cap shale. The borehole trajectory and invasion profile relative to the cap shale are shown (bottom). The model simulates the observed field logs (top), and suggests that the borehole entered a steeply dipping cross-bedded formation below the cap shale, known to exist in the reservoir.
Anisotropy. In highly deviated wells, induction and propagation tools can detect resistivity anisotropy—largely invisible to these tools in vertical wells—because they have been designed to measure currents or fields in planes normal to the tool axis. The origins of electrical anisotropy are linked with the same phenomena that cause permeability anisotropy—formation bedding geometry or grain size in homogeneous sand beds. Often thin, anisotropic, conductive shale beds or laminations are mixed with high-resistivity pay zones.

Although some new properties can be obtained from the use of both horizontal and vertical resistivity, it is horizontal resistivity, $R_h$, that is most desired in log interpretation. In vertical wells, inductive tools read the geometric mean of the horizontal bedding resistivities. In deviated or horizontal wells, it is difficult to define a simple mixing law to represent the way the induction tools read an average of the horizontal and vertical resistivity, $R_v$, which will be very different from the average horizontal resistivity the tool reads in a vertical well.

A version of the 3D FDM model has been developed that can account for any directional variation in resistivity, which will account for any differences in vertical and horizontal resistivities. A simple dipping bed example, with no invasion, shows an induction tool’s responses, logging at a dip of 45° as it moves from an isotropic bed into an anisotropic bed (left). The induction curves all are in good agreement with $R_h$ deep in the isotropic bed, but as the tool moves into the anisotropic bed, all the curves tend towards a weighted average of $R_h$ and $R_v$. In a vertical well, the log curves would all read $R_h$.

To investigate how anisotropy further complicates the already complex interpretation of invasion in a horizontal well, anisotropy was added to the previous example of a horizontal well near a cap shale with invasion. The invaded zone was assumed isotropic; the overall responses are similar to the previous case—polarization horns appear at the bed boundary, and in the invaded zone shallow tool response curves read close to $R_x$ and the induction logs separate—but there is a difference in the deep tool responses. In the anisotropic bed, the deep tool responses tend towards an average of $R_v$ and $R_h$, instead of reading close to $R_h$. Without the model, it would be impossible to determine the presence of anisotropy or accurate formation resistivities based on the behavior of the resistivity curves alone.

Knowledge of vertical and horizontal resistivities is important for analyzing thinly laminated dipping formations, where both resistivity values are crucial for estimating the sand lamina resistivity and the net-to-gross ratio (the sand fraction) simultaneously. Armed with the 3D modeling capability and knowledge of the relative dip, the formation resistivity anisotropy can be determined. Of course, the inverse is also true, if the formation resistivity anisotropy is known, then the relative dip or deviation angle can be derived from the induction tool responses.

Characteristics of typical modeled invasion profile geometries. The simple step invasion profile and the ramp profile require three parameters; the slope profile is a four-parameter model; the annulus profile is a five-parameter model.
The response of propagation tools to anisotropy is even more pronounced than the response of induction tools. At high dip, the vertical component of resistivity for both tools is multiplied by a term proportional to frequency. This term is an order of magnitude greater for 2-MHz propagation tools than for induction tools. The anisotropic response also varies greatly with the transmitter-receiver spacing. The highly nonlinear transforms from phase shift to resistivity and attenuation to resistivity behave differently, amplifying the anisotropic effects when the two logs are compared.

Resistivity Modeling as a Log Analyst’s Tool
Modeling results in the dipping bed and horizontal well examples indicate that resistivity logs in complex formations contain geometrical information, but extracting it is a challenge. At the very least, the use of forward modeling has now become a key tool for log analysts in understanding the formation properties that combine to produce the logging tool responses.

A new workstation program is currently being developed for GeoQuest, called INVASION, to provide log analysts with the tools for inversion-based, resistivity-modeling formation analysis. The programs are based on forward modeling for layered formations, and the interpretation allows for evaluating dynamic reservoir properties—early-time permeability, water cut and fractional fluid flow. The system helps take the drudgery out of the most time-consuming activities associated with modeling. This is done with a graphical interface and interactive parameter selection, which promotes a more accurate $R_h$ evaluation from multiple resistivity measurements (above right).

The AIT induction logs are corrected for environmental effects either at the wellsite or by a preprocessing program called PrePlus, which corrects for apparent dip and effects of shoulders. The resulting logs are vertically matched in resolution. The logs are then processed by a resistivity iterative inversion program, using 1D radial tool response functions as a forward model. The combination of separate vertical processing and radial processing is called 1D+1D processing. This step, along with log squaring, gives the analyst a first approximation of the formation beds and resistivities. At this point, using a graphical log display, the log analyst can review the initial formation model and make refinements to the model with an interactive interface.

This interactive task permits the analyst to manually define invasion resistivity profiles and the formation bedding and resistivity description through a tabular interface (previous page, right). With this interactive task, the analyst can explore the sensitivity of log data to changes in the formation model.

29. Also see Thadani SG and Hall HE Jr: “Propagated Geometrical Factors in Induction Logging,” Transactions of the SPWLA 22nd Annual Logging Symposium, Mexico City, Mexico, June 23-28, 1981, paper WW.
Sensitivity analysis is especially important when $R_{xo}$ and $R_a$ are dissimilar. With the same task, the user can change $R_a$ and invasion profiles (invasion and annulus radii and resistivity $R_{xo}$ and $R_{ann}$) and quickly recompute, using forward modeling, the synthetic log for verification with original logs. For laterolog responses, the forward model uses the 3D FEM for dipping beds and a 2D FEM otherwise. For induction tools, the forward model is based on 2D hybrid FEM, and a 1D analytical code is used for dipping beds. Formation models can include invasion geometry as either a step function, ramped, annulus profile or no invasion.

During the next phase of interpretation, after the analyst is satisfied that the final formation model accurately represents the logging environment, there are two new preliminary functions to compute invaded zone fluid properties. First, with invasion parameters, LWD and wireline resistivities and porosity logs, the volume of mud filtrate that invaded the formation around the borehole can be computed. This allows a time-lapse permeability analysis. Also, if the well is vertical in a clean or shaly sand and has beds thicker than 6 ft [1.8 m], the analyst can compute a formation water-cut log—useful for reservoir engineering, and a fractional-flow log, both based on the fluid dynamics reflected in the formation invasion profile (above).

This graphically interactive, resistivity modeling-based formation evaluation program facilitates interpretations in complex formations.


**Interpreting invasion.** Model-based fractional flow inversion logs enable reservoir engineers to predict water cut throughout the reservoir. With porosity and the fractional flow computed from the invasion profile, a water-cut log is computed that helps determine which part of the reservoir to perforate. In many cases, it may be necessary to minimize water cut, and in other cases a higher water cut may be acceptable for optimum oil production.

**Outlook for Resistivity Modeling**

The recent developments in code efficiency will lead to full use of 3D models for interpretation applications. Induction, propagation, laterolog and eventually other logging tools, such as nuclear and acoustic, will be modeled in more realistic formations. Petrophysical relationships will be incorporated in the models, which means the formation will be described in log analysis terms: lithology, porosity and saturation.

Trends started with AIT and PLATFORM EXPRESS equipment will continue—sophisticated tool environmental corrections will be built into the logs using results from detailed physical and numerical modeling during the tool design and development phases. The power of analysts’ tools based on resistivity modeling, such as in the R_BAN, INFORM and INVASION programs, will be brought to bear on petrophysical interpretation in complicated environments. Many consumers of well log data believe that to gain maximum value from resistivity logs in general, and induction logs in particular, the numerical modeling of tool responses will be an indispensable facet of interpretation for the 21st century.

—RCH