Abstract

The problem cased by wax deposition during production and transportation of waxy oil has been receiving an increasing attention from the industry, especially with the off-shore projects expansion recently. Due to the high cost of those projects, an accurate modelling on wax precipitation and deposition is imperatively required to facilitate the design, development, and operation of those projects. While the wax precipitation models depict the thermodynamic behaviour of waxy oil, the modelling of wax deposition in production facilities and pipelines focuses on how soon wax would accumulate. The wax deposition simulation appears to be a more challenging task due to the high complexity and limited measured data available in the open literature. In this work, a compositional simulator is introduced to simulate the wax deposition in a pipeline. This model is founded on a thermodynamics model from our previous work and equipped with major wax deposition mechanisms as well as tuning function. Through four case studies, the model, with its prediction and tuning capability, has demonstrated a practical tool in flow assurance simulation. The results also suggest that a promising methodology can be applied in flow assurance study when the simulator is tuned with measurements from the high pressure deposition cell.
Introduction
The problem caused by wax deposition during production and transportation of waxy oil has been receiving an increasing attention from the oil and gas industry with the rapid expansion of offshore projects. This problem mainly happens when the oil is cooled to a temperature below wax appearance temperature (WAT) due to lower environment temperature. Under such a condition, wax solid will precipitate out and may accumulate in the pipeline and eventually plug the pipeline. Therefore, an accurate modeling on wax precipitation and deposition is imperatively required to put this hazard situation under control and facilitate the design, development, and operation of offshore projects.

A reliable wax precipitation model provides the thermodynamic properties and the driving forces to wax deposition model and therefore serves as the foundation of flow assurance simulation. A couple of thermodynamic models have been proposed to depict wax precipitation phenomenon in the literature (Pedersen [1], Lira-Galeana et al. [2], Pan et al [3], Coutinho et al [4] and Zuo et al [5] [6]). The Zuo’s model has shown a good agreement with experimental data by adopting a framework with the Peng-Robinson equation of state for describing the nonideality of the vapor and liquid phases and the predictive universal quasi-chemical (UNIQUAC) for the solid (wax) phase. This model has been incorporated into a compositional flow assurance simulator and will be used in this work.

The modeling of wax deposition in production facilities and pipelines focuses on how soon wax would accumulate on the wall of pipeline. Compared to wax precipitation modeling, the wax deposition simulation demonstrates a more challenging task due to the higher complexity and limited measured data available in the open literature. Many researchers (Brown et al [7], Burger et al [8], Svendsen et al [9], Singh et al [10][11], Lindeloof et al [12], Edmonds et al [13], Rygg et al [14], Matzain [15], etc.) have contributed to the modeling of wax deposition in pipeline. Their work revealed that a number of mechanisms, such as, molecular diffusion, shear dispersion, shear reduction, Brownian diffusion, thermophoresis, and internal diffusion, appear to be responsible for the radial wax deposition in a pipeline. However, a quantitative prediction of wax deposition remains to be a challenge.

In this work, an effort has been made to simulate wax deposition in a pipeline using a compositional wax deposition simulator. A brief description of model is given, followed by four case studies. In case study 1, two alternative approaches will be adopted to predict the wax deposition due to molecular diffusion in laminar boundary sublayer, which is widely accepted as dominant mechanism for wax deposition in pipeline. The prediction results are compared against measured flow loop data to justify current model. In case study 2, the model is tuned using wax deposition data scaled up from a high pressure deposition cell measurement. The tuned parameters are used to predict the case from the high pressure deposition cell measurement. In case study 3, one full scale pipeline case is simulated either with or without the tuning parameters obtained from case study 2, and a comparison between the simulation results and data reported from literature is given. In case study 4, a scaling model is used to simulate wax deposition in two phase flow. The results are presented together with a single phase flow simulation. Some conclusions are summarized afterwards.

Model description
In this work, the pipeline is divided into segments over which the heat transfer and pressure drop are calculated. Since the amount of wax deposition is very small relative to the total fluid within a reasonable time step, a steady state flow is assumed. With this assumption, the pressure and temperature profile are obtained from the fluid dynamics and heat transfer modules, and used for further modeling of wax deposition on each segment.

As above mentioned, the modeling of wax deposition is established on a thermodynamic model by Zuo et al. [5] [6]. The basic assumption that wax formers are normal paraffins heavier than nC17 has been inherited in the wax deposition model. Therefore the wax deposition layer in a pipeline may be considered as a gel of normal paraffin solid and entrapped oil. The volume fraction of entrapped oil in deposition layer is defined as porosity, which is usually found within a range of 40% to 90%. The porosity is introduced as an important input parameter for mass transfer calculation as shown below. The mass transfer rate, i.e., the rate of wax deposition, is assumed to be a summation of mass transfer rate due to two major mechanisms, molecular diffusion and shear removal.

1. Molecular diffusion
As the dominant mechanism of wax deposition, molecular diffusion in laminar boundary sublayer is considered in each case study of this work. Driven by the radial concentration gradient in oil phase, the deposition rate of wax formers (n-paraffins) due to molecular diffusion may be calculated by the product of diffusion coefficient and the concentration gradient at the wall as shown below,

\[ J_i = -D_i \left( \frac{dC_i}{dr} \right)_{w} \]  

(1)
where \( i \) refers to each wax former, \( J_i \) is the mass transfer rate (mol/m²/s) due to molecular diffusion of component \( i \), \( D_i \) is the diffusion coefficient (m²/s) and \( \left( \frac{dC_i}{dr} \right)_w \) is the concentration gradient at the wall (1/m).

Two approaches are usually adopted to calculate \( \left( \frac{dC_i}{dr} \right)_w \). The first approach is called heat-mass analogy method. In this method, the driving force is calculated through heat-mass analogy using concentration in bulk fluid and at the wall. The latter is essentially the saturate concentration of wax formers at the wall temperature.

\[
\left( \frac{dC_i}{dr} \right)_w = -\frac{Sh}{2R} \Delta C_i
\]

Here \( Sh \) is the Sherwood number, \( R \) is the pipeline inner radius, and \( \Delta C_i \) is the difference between concentrations in bulk fluid and saturate concentration in oil at the wall temperature.

The second approach is called solubility method, in which \( \left( \frac{dC_i}{dr} \right)_w \) is calculated through temperature gradient at the wall by assuming that wax formers are saturate everywhere in the oil near the wall. In other words, the temperature gradient will determine the concentration gradient at the wall by

\[
\left( \frac{dC_i}{dr} \right)_w = \left( \frac{dC_i^s}{dT} \right)_w \left( \frac{dT}{dr} \right)_w \tag{3}
\]

where \( \left( \frac{dT}{dr} \right)_w \) is the temperature gradient at the wall and \( C_i^s \) is the saturate concentration of wax former \( i \) at temperature \( T \).

A study by Venkatesan et al [16] suggested that the actual concentration profile is located between the profiles from these two approaches. That is, the solubility method may predict a lower limit while the heat-mass analogy may bring an upper limit for actual concentration gradient at the wall.

In addition to the driving force, in this model the diffusion coefficients \( D_i \) of those wax formers (n-paraffins) are approximated by the diffusion coefficients in infinite dilutions (i.e., a solution in which the concentration of component of interest is approaching zero). This approximation can be justified by the fact that the solubility of wax formers in oil tends to be of a low value. Like many other wax deposition models, the popular Hayduk-Minhas method [17] is used for this calculation.

After obtaining the deposition rate of normal paraffins due to molecular diffusion, one need turn to those non-wax formers. It is worth mentioning that although the deposition rate of n-paraffin solid could be purely predicted from eq. (1)-(3), the deposition rate of other components has to be obtained by compromisingly introducing wax porosity as an input parameter. To be more specific, in this work the overall deposition rate due to molecular diffusion is scaled up from the deposition rate of n-paraffins using the given wax porosity. This compromise comes from an implied assumption behind eq. (1) that the molecular diffusion rate of a certain component is proportional to the concentration of this component itself, which is valid only for components with concentration approaching zero. On the other hand, let us suppose that this assumption can be applied to each component. Then if wax deposition exists only the wax formers move towards the wall, while the remaining components move in the opposite direction, since at the wall n-paraffins have concentration lower than concentration in bulk fluid and the remaining components correspondingly have higher due to the precipitation of n-paraffins. This will result in zero porosity, which is contradicted with the observation in experiments.

2. Shear removal

When acting on a wax gel layer, the shear stress may slough off wax pieces from the deposition layer. This effect seems to be the major mechanism to reduce the deposit thickness in most of cases. The negative mass transfer rate due to shear removal \( J^T \) may be related to the shear stress, thickness of wax gel layer, and strength of wax gel yield stress. Kern and Seaton [18] have suggested that this term is proportional to the shear stress and deposit thickness. Recently it has been found by
Venkatesan [19] that the wax gel yield stress might be proportional to \( \omega^{2.3} \), where \( \omega \) is the mass fraction of wax in gel layer. This finding has enabled Edmonds et al [13] to extend Kern Seaton’s work as

\[
J^w = k\delta \tau / \omega^{2.3}
\]

(4)

where \( \delta \) is the deposit thickness, \( \tau \) is the shear stress and \( k \) is a parameter.

In this work, an additional parameter \( a \) has been introduced to eq. (4) for a better fitting function, inspired by Venkatesan’s [19] fitting that \( J^w \) may be proportional to \( \tau^{1.9} \), instead of \( \tau \). Hence \( J^w \) may be expressed by

\[
J^w = k\delta \tau^a / \omega^{2.3}
\]

(5)

For a single phase flow, the mass transfer rates obtained from the above two mechanisms can be converted directly to the wax deposition thickness. But the same strategy can not be applied to a two-phase flow region directly, since the wax deposition in a two-phase flow may only occur at the oil wet interface and depend on several two-phase flow characteristics. Researchers such as Rygg et al. [14] or Matzain et al. [15], have suggested that in addition to appropriate two-phase fluid dynamics and heat transfer models, a scaling method, which incorporates characteristic parameters of two-phase flow, has to been applied to scale wax deposition in a two-phase flow from a single-phase model. Following their strategy, a first approximation has been taken in this work and a proposal was made that the wax deposition in two-phase flow is related to liquid holdup and a ratio of wetting surface by

\[
J^{wp} = \Phi^a H^\beta J^{wp}
\]

(6)

Here \( J^{wp} \) is the mass transfer rate using single phase deposition model, \( H \) is the liquid holdup, \( \Phi \) is the ratio of wetting surface, \( \alpha \) and \( \beta \) are tuning parameters with a default value 0.5.

**Results and Discussion**

In this work, the model described above has been applied to four case studies. The first two cases are based on lab scale measurements and the last two cases are given for full scale pipeline simulations.

**CASE STUDY 1**

Hernandez [20] conducted an investigation on the wax deposition under single-phase flow conditions using the flow loops from the University of Tulsa. Two fluids, Garden Banks and South Pelto, were used to study the effects of the flow rate and the temperature difference on the wax deposition. Based on the fluid and flow loops information given in ref. [20], a couple of simulations have been performed using the proposed model. In these simulations, only the molecular diffusion mechanism is taken into account without any tuning, whereas the shear removal term is not considered. Two alternative approaches, heat mass analogy method and solubility method, have been applied.

The simulation results are given in figs. (1)-(6). Figs. (1)-(3) compare the simulation results to the measured data from tests for Garden Banks fluid, while Figs (4)-(6) give the comparison for South Pelto fluid. In these figures, markers represent the measured data, solid lines give the results from simulation using the heat-mass analogy method, and dash-dot lines refer to the simulation using the solubility method. Different markers indicate the deposition thickness measured by different methods. The figures (1)-(6) show that the proposed model provide a fare agreement with the experimental data. The simulations using the heat-mass analogy method generally provide better predictions than the solubility method. Figs. (4)-(6) show that for South Pelto fluid the heat-mass analogy method overpredicts the wax deposition, while the solubility method tends to underpredict the deposition. This observation has confirmed the theoretical analysis given by Venkatesan et al [16]. In most cases of Garden Banks fluid, as shown in Figs. (1)-(3), the simulation results using heat-mass analogy lie among data measured by different methods. This slight inconsistence with Venkatesan’ analysis might be caused by the deviation from the thermodynamics or the transport property models. It might also be a result from uncertainties in the flow loop measurements.

**CASE STUDY 2**

The experimental results given in case 1 have suggested that the measurement of deposition thickness in flow loop test could have significant discrepancies between results from the different measurement methods or from the different runs under the same condition. This situation may come from the difficulties in maintaining an ideal operation condition for flow loops. These discrepancies bring a non-negligible uncertainness if the data are used for parameter tuning. Meanwhile, another tool, the high pressure deposition cell has shown a promising performance in the study of wax deposition by scaling the cell data to a pipeline segment [21]. Since the measured data from high pressure deposition are obtained with a relative small discrepancy, these data may provide a good source to demonstrate the tuning function of the proposed model.
In the proposed model, the overall deposition rate can be given as,

\[ J^{overall} = C_1 \sum_i J_i - C_2 J^{sr} = -C_1 \sum_i D_i \left( \frac{dC_i}{dr} \right)_w - C_2 \delta \tau^a / \omega^{2.3} \]  \hspace{1cm} (7)

Eq. (7) implies that three tuning parameters, \( C_1 \), \( C_2 \) and \( a \) may be used for tuning. However, as three data points are provided in ref. [21], only two of the three parameters can be used for the tuning. Following Venkatesan’s suggestion, \( J^{sr} \) is proportional to \( \tau^{1.9} \), parameter \( a \) has been fixed to be 1.9 and parameters, \( C_1, C_2 \) are open for tuning.

This case study has been conducted using both the heat-mass analogy and the solubility methods for molecular diffusion using information given in Table 6 of ref. [21]. The simulation and tuning results are given in Table 1.

<table>
<thead>
<tr>
<th>Test number</th>
<th>Molecular diffusion model</th>
<th>Measured deposition thickness (mm)</th>
<th>Simulated deposition thickness (mm)</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPS-1</td>
<td>Heat-mass analogy</td>
<td>0.037</td>
<td>0.037</td>
<td>1</td>
<td>6.6E-4</td>
</tr>
<tr>
<td></td>
<td>Solubility</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EPS-2</td>
<td>Heat-mass analogy</td>
<td>0.013</td>
<td>0.013</td>
<td>1</td>
<td>6.6E-4</td>
</tr>
<tr>
<td></td>
<td>Solubility</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EPS-3</td>
<td>Heat-mass analogy</td>
<td>0.004</td>
<td>0.00354</td>
<td>1</td>
<td>6.6E-4</td>
</tr>
<tr>
<td></td>
<td>Solubility</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The tuning starts from parameter \( C_1 \) based on test 1 of equivalent pipe section (EPS-1), in which the shear stress is small. By neglecting the shear removal term, it has been found that no tuning is needed for molecular diffusion term when the heat-mass analogy is adopted. On the other hand, a multiplier 1.5 has to be applied while using the solubility method. This seems to be reasonable if we compare with the results obtained in Case 1. After obtaining the optimized value of \( C_1 \), test ESP-2 is used to tune the value of \( C_2 \). For both heat-mass analogy and solubility methods, 6.6E-4 seems to be an optimum value for \( C_2 \). With this value, EPS-1 has been re-visited by taking into account both molecular diffusion and shear removal effect. A negligible change has been observed in this round. Therefore no iteration is needed to tune the parameters \( C_1 \) and \( C_2 \). These parameters are directly used for simulations of test ESP-3. A comparison with experimental data can be found in the last two rows of Table 1. It is shown that the proposed model and the tuning method gives good predictions for test ESP-3. The results suggest that the simulation model combined with results from the high pressure deposition may provide a reliable tool to study wax deposition in real pipeline and optimize the design of offshore projects.

**CASE STUDY 3**

After performing simulations of wax deposition against the lab-scale measurements in case studies 1 and 2, a case of full scale pipeline was studied, which was reported as case study 1 by Brown et al. in ref. [7]. The first run of simulation was performed without any tuning. Only molecular diffusion is considered by using the heat-mass analogy. The results from this simulation are given in figure 7. The total deposited wax volume has been obtained around 1,200 bbl. This value is close to the results from other simulations, 1,070 bbl by Brown et al [7] or 995 bbl by Edmonds et al [13], but is much higher than the data reported from field, which is “more than 50 bbl”. This overprediction suggested that the shear removal effect may not be neglected for this case. The simulation was then repeated by considering both molecular diffusion and shear removal mechanisms. Due to insufficient information from the literature, a customized tuning seems to be impossible, and the tuned parameter in Case 2 was tentatively used. Figure 8 gives the results from the second run of simulation. The results show that a balance between molecular diffusion and shear removal has been achieved after about 80 days. And the total deposition wax volume was about 25 bbl. It is no surprise that this value does not agree very well with the measured “more than 50 bbl” since the totally different oil and pipeline configuration was used to obtain the tuning parameters. Generally speaking, this simulation has confirmed the significance of shear removal in the wax deposition predictions.

**CASE STUDY 4**

In this case, another full scale pipeline simulation has been conducted to deal with wax deposition in two-phase flow. The oil and pipeline information have been reported by Lindeloff and Krejbjerg in [12]. The simulation has mimicked two flow assurance scenarios with only molecular diffusion considered by applying the heat-mass analogy method. In the first scenario, the pipeline is insulated by concrete and surrounded by sea water. And the inlet pressure is as high as 100 atm to ensure a single phase flow. The simulation result is given in figure 9. It is shown that the deposition reach a peak near the
pipeline inlet with oil temperature quickly goes below wax appearance temperature. Further down the pipe, the temperature of oil continues going down and wax deposition becomes less significant. A comparison was made with another two simulations by Lindeloff and Krejbjerg [12] and by Edmonds et al [13]. The simulation in this work seems to have a similar shape of wax deposition layer with [12] but the deposition thickness is closer to that from [13]. Generally speaking the three simulations provide a similar prediction. In the second scenario, the pipeline is buried in the soil and a two-phase flow is induced by applying an inlet pressure of 30 atm. At the end of pipeline is a riser surrounded by water. The method from eq. (6) is used for the two-phase flow scale-up. Figure 10 shows the simulation result. Since the thermal resistance of soil is much higher than that of water, the wax deposition occurs at a location further down the pipeline compared to the first scenario. After that, the thickness of deposition layer becomes stable with a low heat loss. Finally, the wax deposition rate increases rapidly due to a much higher heat loss when oil entering the riser. This has been again compared with the results given in [12] and [13]. Again, the shape of wax deposition layer is quite similar to that in [12], while the deposition thickness seems close to the result from [13].

Conclusions
A compositional model for the simulation of wax deposition in pipelines has been proposed with tuning methods in this work. Founded on an in-house thermodynamic module, this one dimensional, pseudo-steady simulator demonstrates a simple and powerful tool in flow assurance simulation. Through case studies 1-3, the current model provides reasonable predictions and tuning capabilities in single-phase flow simulation. In particular, the results suggest that a promising methodology can be applied in flow assurance study when the simulator is tuned with measurements from the high pressure deposition cell. The case study 4 reveals that wax deposition in a two-phase flow is lower than that in a single-phase flow. The proposed model has demonstrated a generally similar prediction performance when compared with other two simulators for both single-phase and two-phase flow.

References
Garden Banks WAX2001-025 (Q=1500 BPD, $\Delta T=30$ F)

Figure 1 Flow loop test for Garden Banks fluid, flow rate = 1500 BPD, $\Delta T=30$ F
Figure 2 Flow loop test for Garden Banks fluid, flow rate = 1500 BPD, $\Delta T$=15 F
Figure 3 Flow loop test for Garden Banks fluid, flow rate = 1800 BPD, ΔT=30 F
Figure 4 Flow loop test for South Pelto fluid, flow rate = 1800 BPD, ΔT=45 F
Figure 5 Flow loop test for South Pelto fluid, flow rate = 2900 BPD, ΔT=45 F
Figure 6 Flow loop test for South Pelto fluid, flow rate = 4500 BPD, ΔT=45°F.
Figure 7 Simulation of wax deposition along the pipeline in case study 3, only molecular diffusion applied
Figure 8 Simulation of wax deposition along the pipeline in case study 3, molecular diffusion and shear removal applied
Figure 9 Simulation of wax deposition along the unburied pipeline in case study 4, single phase flow
Figure 10 Simulation of wax deposition along the buried pipeline in case study 4, two phase flow