**Semblance Criterion Modification to Incorporate Signal Energy Threshold**

*Sandip Bose*, Henri-Pierre Valero and Alain Dumont, Schlumberger Oilfield Services

**SUMMARY**

The semblance criterion widely used for slowness estimation is examined using the theory of statistical hypothesis testing and shown to be equivalent to that for detecting propagating signals of any energy at a specified slowness. We propose to suppress weak collar arrivals on semblance maps for logging while drilling (LWD) data by incorporating a minimum signal energy threshold in the aforementioned theory and using it to derive a couple of modifications of the semblance. Other simpler modification inspired by these are also proposed on heuristic grounds. All proposed modifications perform comparably on real LWD data and are effective at suppressing weak unwanted arrivals while preserving the desired ones.

**INTRODUCTION**

The semblance criterion is the basis for a widely used method (Kimball and Marzetta, 1984) for estimating sonic slowness especially with compressional and shear headwave (P&S) logging. This is an array-based non-dispersive processing method suitable for detecting signals irrespective of their energy. This property is invaluable for detecting compressional arrivals that are usually weak relative to other arrivals and accurately extracting their slowness. For this reason it has been extremely successful and widely used.

However a side effect of the same invariance of semblance to signal amplitude is that it also responds to very weak events such as weak tool or casing arrivals. While methods exist to handle such unwanted semblance peaks, these become a more serious issue for LWD applications where the processing has to be conducted downhole. Even when the sonic hardware is designed to attenuate the tool arrivals and further mitigation is possible with advanced processing techniques, a need may still exist to avoid such spurious semblance peaks.

We propose to address this issue by requiring that the signal energy exceed a given threshold so as to suppress semblance output on weaker collar arrivals. This requires a modification of the semblance criterion. One approach is based on interpreting semblance as a test statistic for detecting coherent arrivals of any energy, as explained below, and modifying it to correspond to the test statistic for detecting signals satisfying the energy threshold requirement. This approach leads to two new candidate modifications based on imposing the signal threshold in two different ways. Another approach is to use heuristic arguments to obtain simpler modifications partly inspired by the first approach; i.e. by thresholding the coherent energy and by subtracting the threshold from both coherent and incoherent energy, we propose two more candidate modifications. We then examine the performance of each of these modified criterions and present the results from comparing all the proposals on real LWD data in subsequent sections.

**REVIEW OF ORIGINAL SEMBLANCE CRITERION**

We begin by reviewing the classical semblance criterion as proposed in Kimball and Marzetta (1984) and how it is used in estimating slowness. Recall that given an array of waveforms, $x_l(t), l=1,\ldots,L$, we proceed by placing windows of specified length $T_w$ at time locations and moveouts given by $\tau$ and $p$ respectively, and computing the semblance criterion given by

$$\rho(\tau, p) = \frac{\int_{\tau}^{\tau+T_w} \sum_{l=1}^{L} x_l(t+p\delta_l)^2 dt}{\int_{\tau}^{\tau+T_w} \sum_{l=1}^{L} |x_l(t+p\delta_l)|^2 dt}$$  \hspace{1cm} (1)$$

for each of these windows. The moveout corresponding to the peaks of the semblance criterion are then declared to be the slowness of the non-dispersive components in the received data.

For discrete time sampled systems, the integrals in equation (1) are replaced by sums over corresponding windows, i.e.

$$\rho(\tau, p) = \frac{\sum_{n=1}^{\delta} \sum_{l=1}^{L} |D_{\delta_l}x_l(n)|^2}{\sum_{n=1}^{\delta} \sum_{l=1}^{L} |D_{\delta_l}x_l(n)|^2}$$  \hspace{1cm} (2)$$

where $D_{\delta_l}$ is a time-shift operator that shifts the input by $\delta_l$ (need not be a multiple of the time- sampling period).

This criterion has been studied (Douce and Laster, 1979) and is widely used in processing non-dispersive arrivals. It has been used to successfully identify arrivals irrespective of amplitude. It is therefore of interest to revisit the problem for which this criterion happens to be the optimum solution.

**Signal detection problem**

We now show that the semblance criterion is nothing but the likelihood ratio test statistic (Lehmann, 1986) for a detection (hypothesis testing) problem. To see this, let us consider the signal detection problem for the case where we observe an array data $Y_{L\times N_0}$ comprising $N_0$-length traces collected at $L$ receivers and try to detect if a common (but unknown) signal $s$ is present in all receivers.

In other words, we have the problem of testing between the following two hypotheses about the data

$$H_0 \ : \ Y = \mathcal{N} \ \ \ \text{vs.} \ \ H_1 \ : \ Y = \mathbf{1} \cdot s + \mathcal{N},$$  \hspace{1cm} (3)$$

where $\mathbf{1}$ is a column vector of all 1’s, and used to indicate that the same signal trace $s$ is present in all receivers under hypothesis $H_1$. $\mathcal{N}$ represents the noise that is assumed to follow the white Gaussian distribution with unknown variance $\sigma^2$.

This hypothesis testing problem can be solved by computing the Generalized Likelihood Ratio Test (GLRT) (Lehmann, 1986; Van Trees, 1968) statistic and comparing it to a threshold. The former is obtained by computing the likelihood function1 under each hypothesis and taking the ratio of its maximized value

---

1The likelihood function is obtained from the probability model for the observed data.
PROPOSALS FOR MODIFICATION OF SEMBLANCE

We now turn to addressing the issue (particularly for LWD) resulting from the same amplitude invariance. Specifically, in some cases, weak undesired arrivals such as tool or casing (even after mitigation steps in hardware and/or pre-processing) can register as high semblance events thereby masking or confusing the downhole processing of the true arrivals.

Using the insight discussed in the previous section, we can address this issue by requiring a minimum energy threshold for the signal to be detected. In the following sections, we consider two different ways to incorporate this requirement in the signal detection problem and in each case, derive a suitable modification to the detection test statistic and therefore the semblance.

Detection of signal above threshold

We once again set up the detection problem as a hypothesis testing problem as before but with the additional requirement that the signal present under $H_1$ meets some specified threshold on its energy (or amplitude):

$$H_0 : Y = \mathcal{N} \quad \text{vs.} \quad H_1 : Y = \frac{1}{\varepsilon} \cdot \mathcal{N}, \quad \|y\|^2 \geq \varepsilon^2,$$

where now we have imposed a threshold $\varepsilon^2$ on the energy of the unknown signal.

The maximized log likelihood under $H_0$ is identical to that of the previous section. The quantity under now has to be maximized under the constraint in (10) on the signal norm. This yields the signal estimate

$$\hat{s} = x_1 \frac{Y}{\|y\|}, \quad \hat{s}_1 = \max \left( \varepsilon, \frac{\|Y\|^2}{L} \right).$$

Using this estimate and repeating the same steps as in the previous section, we obtain the GLRT statistic for this problem as

$$t_{GLRT} = \frac{N_0 L}{2} \log \frac{\|y\|^2}{F} = \frac{N_0 L}{2} \log \frac{1}{1 - \rho},$$

where we have used $|P_Y^{-1}|^2 = \|Y\|^2/2$ and defined

$$\rho = \frac{|P_Y^{-1}|^2}{\|y\|^2} = \frac{1}{L} \sum_n \nabla y_{mn}^2 / L \|y\|^2 = \sum_n \sum_m |y_{mn}|^2 / L \|y\|^2.$$

We observe that the last quantity $\rho$ has exactly the same form as the semblance of equation (2) used in non-dispersive processing. Since the GLRT is a monotonic function of the semblance $\rho$, we hold the latter to be equivalent to the former for the purpose of detecting a signal present in all the sensors. Therefore, we can interpret our slowness processing methodology as consisting of running a detector for each of a number of time-window locations and moveouts and estimating the slowness of propagating non-dispersive components as those values of the moveout where the detector output shows a local peak. We note that the semblance criterion is invariant to any scaling of the data and therefore is effective in detecting weak arrivals such as the compressional even with widely varying amplitudes. Hence the semblance criterion is widely used in commercial processing.

For proposals to modify the semblance is used for the purpose of detecting a signal present in the data.

Rejection of signal below threshold

We now look at a more general scenario where we also invoke a threshold under $H_0$; i.e., we expect that a signal could be present below a threshold but interpret it as a spurious arrival rather than the desired signal. Of course, we also need to have a threshold to declare signal presence and the latter threshold must be greater than the former.

In other words, we consider the detection problem as before but with the following modifications:

$$H_0 : Y = \frac{1}{\varepsilon} \cdot \mathcal{N}, \quad \|y\|^2 \leq \varepsilon_0^2$$

vs. $H_1 : Y = \frac{1}{\varepsilon_1} \cdot \mathcal{N}, \quad \|y\|^2 \geq \varepsilon_1^2,$

where now we consider the signal energy to obey thresholds $\varepsilon_0$ and $\varepsilon_1$ under $H_0$ and $H_1$ respectively with $\varepsilon_1 \geq \varepsilon_0$. 

It is simply the probability density function evaluated at the observed value expressed as a function of the parameters of the probability model. Thus when we have a observable $X$ with a probability density function $f_{X|x}$ from a model parameterized by $\theta$, we can write the likelihood function for a given observed value $x$ as $L(\theta|x) = f_{X|x}(x)$. 

We obtain $\max L(\theta|x, H_1) = \max L(\theta|x, H_0) = \max L(\theta|x, H_1)$ as the test statistic to be compared to a threshold.

In our problem, we compute the log likelihood function under $H_1$ using the assumption of white Gaussian noise; i.e.,

$$LL(\sigma^2, s|Y; H_1) = K - \frac{N_0 L}{2} \log \frac{1}{\sigma^2} - \frac{1}{2\sigma^2} \|Y - 1 \cdot s\|^2_F,$$

where $\|p\|^2$ refers to the Frobenius norm of the argument and $K = -\frac{N_0 L}{2} \log(2\pi)$ is a constant. The $LL$ is maximized with respect to $s$ by decomposing the quantity inside the Frobenius norm as $\|P_{1}^\perp Y\|^2_F + \|P_{1}^\parallel Y - 1 \cdot s\|^2_F$ where $P_1^\perp = \frac{1}{L-1} \cdot Y$ and $P_1^\parallel$ are the projection operators onto the subspace of $\perp$ and its orthogonal complement respectively. This maximization removes the second term in the decomposition above and finally, maximization with respect to $\sigma^2$ yields

$$\max L(\theta|x, H_1) = K - \frac{N_0 L}{2} \log \frac{\|P_{1}^\perp Y\|^2_F}{N_0 L} - \frac{N_0 L}{2}.$$

Carrying out a similar development for the log likelihood under $H_0$, we obtain

$$\max L(\theta|x, H_0) = K - \frac{N_0 L}{2} \log \frac{\|Y\|^2_F}{N_0 L} - \frac{N_0 L}{2}.$$

Therefore, we can now obtain the GLRT statistic by taking the difference of equation (6) and equation (7) and canceling the common terms,

$$t_{GLRT} = \frac{N_0 L}{2} \log \frac{\|Y\|^2_F}{\|P_{1}^\perp Y\|^2_F} = \frac{N_0 L}{2} \log \frac{1}{1 - \rho},$$

where we have used $\|P_{1}^\perp Y\|^2_F = \|Y\|^2_F - \|P_{1}^\parallel Y\|^2_F$ and defined

$$\rho = \frac{\|P_{1}^\perp Y\|^2_F}{\|Y\|^2_F} = \frac{\|Y\|^2_F - \|P_{1}^\parallel Y\|^2_F}{\|Y\|^2_F} = \sum_n \sum_m |y_{mn}|^2 / L \|y\|^2.$$

We observe that the last quantity $\rho$ has exactly the same form as the semblance of equation (2) used in non-dispersive processing. Since the GLRT is a monotonic function of the semblance $\rho$, we hold the latter to be equivalent to the former for the purpose of detecting a signal present in all the sensors. Therefore, we can interpret our slowness processing methodology as consisting of running a detector for each of a number of time-window locations and moveouts and estimating the slowness of propagating non-dispersive components as those values of the moveout where the detector output shows a local peak. We note that the semblance criterion is invariant to any scaling of the data and therefore is effective in detecting weak arrivals such as the compressional even with widely varying amplitudes. Hence the semblance criterion is widely used in commercial processing.
Thus we get the following form:

\[ t_{GLRT} = \frac{N_0 L}{2} \log \frac{1}{1 - \hat{\rho}}, \]

where

\[ \hat{\rho} = \frac{v_1 - v_0}{L\|Y\|_F^2 - v_0}. \tag{14} \]

with

\[ v_1 = \|I' \cdot Y\|^2 - \max(0, L\epsilon_1 - \|I' \cdot Y\|^2) \]
\[ v_0 = \|I' \cdot Y\|^2 - \max(0, \|I' \cdot Y\| - L\epsilon_0)^2. \tag{15} \]

**Discussion and other proposals**

We now examine the behavior of the new semblance criterions proposed above. First, note that the modified criterion of equation (13) equals the standard semblance quantity when the signal estimate exceeds the threshold, i.e., \( \|I' \cdot Y\| > L\epsilon \), and we have absolutely no difference from the standard output. However when the signal norm is below the threshold, the semblance drops rapidly towards zero equaling it at half the threshold value. Below that it turns negative, but that can be ignored as it implies that the presence of signal above the desired threshold is extremely unlikely and in practice we saturate it at zero.

**Thresholded semblance**

The above property of a likelihood ratio detector motivates a much simpler modification of the semblance where we simply apply a thresholding function to the coherent energy and compute the corresponding semblance; i.e.,

\[ \rho_r = \frac{(T_r(\|I' \cdot Y\|))^2}{L\|Y\|_F^2}, \tag{16} \]

where \( T_r(t) = t \) if \( t \geq \tau; \) 0, \( t < \tau \). In other words, we consider the coherent energy only if it exceeds the stated threshold while computing the semblance and call this modification the thresholded semblance. Clearly, this exactly equals the original semblance when the coherent energy exceeds the threshold.

**Subtracted semblance**

A second possible modification is inspired by the form of equation (14) and suggests that we subtract the energy threshold from both the coherent and total energy to correspond to rejection of any signals present below the threshold.

Thus we get the following form:

\[ \rho_\delta = \frac{\max(0, \|I' \cdot Y\|^2 - (L\epsilon)^2)}{\max(\delta, L\|Y\|_F^2 - (L\epsilon)^2)}, \tag{17} \]

where we have thresholded the quantities to keep everything positive and have set a small positive minimum \( \delta \) to keep the semblance stable.

The modified semblance \( \hat{\rho} \) is a general expression, reducing to the single threshold case \( \rho \) when \( \epsilon_0 = 0 \) and to the original semblance when \( \epsilon_1 = 0 \). However, we note that while \( v_1 \) reduces to \( \|I' \cdot Y\|^2 \) when the signal estimate exceeds the threshold \( \epsilon_1 \), the same is not true for \( v_0 \); therefore, \( \hat{\rho} \) is never exactly equal to the original semblance. This bias however becomes small as the signal amplitude increases well above the threshold, the extent of the bias being dependent on the original semblance value. The semblance \( \rho_\delta \) exhibits similar behavior.

We now look at the slowness estimation accuracy using the modified criterions and compare with the original. We run 10000 Monte Carlo trials with added noise at a signal to noise ratio (S/N) of 5dB to tabulate the deviation of the semblance peak slowness from the true value and show the results in Figure 1. The results for the modified semblance \( \hat{\rho}, \hat{\rho}_r, \rho_\delta, \) and \( \rho_r \) are shown labeled as ‘LR’, ‘LR2’, ‘Sub’, and ‘Thr’, respectively. We observe that the error distribution is virtually identical for all cases and we do not compromise the slowness measurement by using the modification to suppress the weak signal.

**Figure 1** The slowness estimation error probability distribution at an S/N of 5dB for the original and each of the modified criterions.

We can in fact further minimize the impact of the threshold by customizing it to the slowness-time region where we expect the unwanted arrivals. For example, if the unwanted signal is a weak collar arrival, we may know its approximate slowness and customize the threshold around it. Figure 2 shows such a case where we have restricted the threshold to the region around 60 \( \mu s/ft \) to suppress an LWD collar arrival. If we can predict the arrival time as well, we might also be able to restrict the threshold in the time domain.
RESULTS COMPARISON

We next evaluate the proposed modifications on a section of LWD field data where the formation is fast and the compressional comes close to the tool arrival. We apply these to the data after filtering it in the passband of [10 16] kHz and show the results in Figure 3 using a composite plot. On the left, we show the slowness projection logs obtained using the standard and the four modified semblance criterions proposed here. On the right we show six panels comprising the waveform plot and the five corresponding slowness time coherence (STC) contour plots for a specific depth frame indicated by the white bar on the log panels. Based on past observations of the tool amplitude, we chose the threshold for each modified criterion to result in null semblance output at a signal level of 1 Pa. In addition the thresholds were customized around the expected collar slowness as shown in Figure 2. The efficacy of all the proposed modifications at suppressing the unwanted collar arrival while accurately extracting the compressional is apparent.

CONCLUSIONS

In this note we present a number of modified semblance criterions based on the approach of setting a threshold on the signal energy. The first criterion is derived by posing the problem as that of detecting a signal with energy (or amplitude) greater than the specified threshold and deriving the generalized likelihood ratio test statistic. The second criterion is derived using the same method by posing the problem as that of rejecting any signal with energy below a specified threshold and detecting it if its energy is above a second threshold greater than the first. These criteria modify the original semblance criterion shown to be equivalent to the GLRT test statistic for detecting a signal of any amplitude. In addition, we also propose a couple of simpler modifications inspired by the form of these likelihood ratio detectors. Tests on real and synthetic data illustrate the effectiveness of all these modifications that perform comparably well at suppressing unwanted arrivals while accurately processing the desired signals.

Figure 3: Comparison example of the original and modified STC processing on LWD field data. Observe the collar arrival in the original semblance processing and how effectively it is removed by the modified semblance criterions with no impact on the main arrival. In fact, when the latter comes close to the collar arrival as shown in this frame, the estimation is actually enhanced.
REFERENCES