G026

Bandwidth Optimization for Compact Fourier Interpolation

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SUMMARY

In this paper, we present a bandwidth-optimization technique for Compact Fourier Interpolation (COMFI). COMFI is a minimum mean-square-error interpolation technique for data sampled at irregular locations. A particular application of interpolation is the estimation of data on a global, Cartesian grid (regularization). The sampling space can be of arbitrary dimension. The interpolated data are computed as a weighted sum of the actual data in a neighbourhood of the selected interpolation locations. The interpolation operator depends on the actual sampling locations and the interpolation location, but not on the data themselves. The operator is designed to have the minimum mean-square interpolation error among all linear operators over a suitable class of spatially band-limited basis functions. We consider, however, the spatial bandwidth as a parameter to be selected in an optimum way. Typically, we compute the COMFI operator to have the largest bandwidth that the actual sampling regime supports, in the sense that the mean-square interpolation error is below an acceptable threshold. We show that the quality of the operator can be strongly dependent on the bandwidth used for its design, and that the optimum bandwidth can vary significantly with spatial location.
Introduction

We consider the general interpolation problem for which we have data sampled at actual (usually irregular) locations, and wish to estimate data at other locations. An example of this problem in seismic data processing is that of common-offset interpolation. In this case, the objective is to alter the midpoint sampling of each common-offset subset of the complete dataset, generally as a preprocessing step that improves the performance of a later algorithm, such as migration. A special case of the general interpolation problem is that of regularization, for which the interpolation locations form a regular, Cartesian grid.

Our contribution to this subject is the extension of an existing algorithm in order to improve its performance, especially in the field of seismic data processing. The following section gives a brief history of the existing algorithm and an overview of the extensions. This is followed by some of the mathematical details. After that, synthetic and real data examples are used to illustrate the process.

Interpolation with bandwidth optimization

The minimum mean-square-error (MSE) interpolation operator for 1D sampling was introduced by Yen (1956), and discussed for use in seismic interpolation by Hale (1980). These operators are designed to be optimum for a fixed spatial bandwidth. Chen and Allebach (1987) published a 2D version, together with an error analysis. The approach is readily extended to arbitrary dimensions, as shown later. Our enhancements to this method for the design of interpolation operators are the following:

1. Treatment of the spatial bandwidth as a parameter. This allows a “wavenumber scanning” technique to compute operators with the maximum spatial bandwidth such that the mean-square interpolation error remains below a user-specified threshold. This enhancement is particularly important when the actual sampling varies spatially, as is often the case with seismic data.

2. Definition of a quality control measure. This measure is the mean-square interpolation error at each wavenumber, and predicts the performance of the interpolation operator as a function of wavenumber.

We will refer to the enhanced algorithm (Figure 1) as “compact Fourier interpolation with bandwidth optimization” (COMFI).

![Flowchart](image-url)

Figure 1: Flowchart outlining the interpolation process. The bandwidth optimization component (“wavenumber scanning”) is highlighted in red.
Minimum mean-square-error interpolation operator

The minimum MSE interpolation operator, \( \mathbf{w} \), can be computed from the following matrix/vector equation:

\[
\mathbf{w}(\mathbf{k}) = \mathbf{S}^{-1}(\mathbf{k}) \mathbf{r}(\mathbf{k})
\]  

(1)

The elements of \( \mathbf{w} \) are the weights that are applied to the acquired data in order to compute the interpolated data. The operator’s dependence on the bandwidth used for its design is shown explicitly through the maximum wavenumber vector, \( \mathbf{k} \). The components of \( \mathbf{k} \) are the maximum wavenumbers in each of the \( N_g \) directions in which the interpolation takes place. Each component has physical dimension inverse to the dimension used for location in that direction.

The sampling location matrix, \( \mathbf{S} \), is a product of sinc functions, and depends only on differences between actual sampling locations, \( \{x_i: i=1,\ldots,M\} \). The number of actual traces used in the interpolation, \( M \), may vary with interpolation location. \( \mathbf{S} \) is a square matrix of size \( M \), given by:

\[
S(k) = (s_{i,j}), \quad s_{i,j} = \prod_{m=1}^{N_g} \frac{\sin(2\pi k_m (x_{i,m} - x_{j,m}))}{2\pi k_m (x_{i,m} - x_{j,m})} \quad 1 \leq i, j \leq M
\]  

(2)

Similarly, the interpolation vector, \( \mathbf{r} \), is a product of sinc functions, and depends only on differences between the actual sampling locations and the interpolation location, \( \mathbf{y} \), as follows:

\[
r(k) = (r_1,\ldots,r_M), \quad r_j = \prod_{m=1}^{N_g} \frac{\sin(2\pi k_m (x_{i,m} - y_{j,m}))}{2\pi k_m (x_{i,m} - y_{j,m})}
\]  

(3)

The mean-square error of any candidate interpolation operator, relative to a basis of cosine functions normalized to have unit rms amplitude, and averaged over both wavenumber and phase, is

\[
MSE = 1 - \frac{1}{2} \sum_{i=1}^{M} w_i r_i + \sum_{i,j}^{M} w_i w_j s_{i,j}
\]  

(4)

The minimum mean-square error is

\[
MMSE = 1 - \sum_{i=1}^{M} w_i r_i
\]  

(5)

and corresponds to the operator given by equation 1. In our enhanced scheme, the maximum wavenumber, \( \mathbf{k} \), is given a maximal value which is consistent with keeping \( MMSE \) below a user-specified threshold.

Example interpolation operator

Figure 2 (left panel) depicts an example of a 2D data interpolation problem. The objective is to interpolate data sampled at the actual, irregular locations (blue stars) to the interpolation location (red diamond). The number of actual sampling locations is 25, numbered for reference as indicated in the figure. The numbering scheme is arbitrary, and different schemes just result in permutations of the entries in the matrices involved. For this example, the data irregularities are not severe, and the average sample interval in each direction is 12.5 m.

The computed interpolation operator is displayed in the right panel of Figure 2. The maximum wavenumber for this operator was 0.04 m\(^{-1}\) in both directions. This wavenumber is the Nyquist corresponding to the average sample interval. As expected, the operator gives larger weights to the actual sampling locations close to the interpolation location.
The corresponding mean-square interpolation error spectrum is displayed in Figure 3. Dark blue colors represent very small interpolation errors below 1%. The errors are generally small because the irregularities are minor, and the maximum wavenumber used in the operator design is appropriate for the actual sampling. The errors are lower along the inline wavenumber direction because some of the actual data locations are close to the inline through the interpolation location. The error spectrum shows that the operator will interpolate accurately all wavenumbers within its design bandwidth except for crossline wavenumbers above about 0.03 m⁻¹.

The MSE used to optimize the design bandwidth is typically the rms average of the spectrum shown in Figure 3. If this MSE is lower than the threshold, then the operator will be accepted, otherwise the maximum wavenumber will be reduced and a new operator computed. Acceptance criteria other than the MSE can be used if desired; for example the maximum error in the spectrum may be considered important.

**Application to exploration seismic data**

In this section, we give an example of the application of the new technique to exploration seismic data, which are often irregularly sampled along spatial coordinates. A typical problem is the reconstruction of the irregularly sampled data on a regular, Cartesian grid. Hindriks and Duijndam (2000) proposed the use of least-squares estimation of the Fourier components for data reconstruction, consisting of a transformation of the actual data into the wavenumber domain, and correcting there for the distortions due to the irregular sampling. The reconstruction of the data in the space domain is finally achieved by an inverse Fourier transform onto the desired Cartesian grid. Reconstruction techniques like the one we propose in this paper have not been widely-used with seismic data. This is probably because spatial variations in the actual sampling make most localized schemes difficult to parameterize in a
way that is optimum for the whole survey area. Camargo and Santos (2005) suggested the use of Rosenfeld's (2002) block uniform re-sampling technique, which can be interpreted as a space-domain version of the Fourier reconstruction method mentioned above. Our approach is more direct in that it gives the reconstruction operator with minimum mean-square error, whilst still considering the data to be composed of Fourier components through the choice of basis functions.

The fold map of a common-offset from a 3D marine seismic data set is shown in Figure 4. Some bins are empty (white), most contain a single trace (light green), and some contain two or more traces (dark green). Figure 5 shows an inline (arrowed in Figure 4) from the irregular data set (left panel) and after regularization (right panel). This particular line has many empty bins, which are populated with accurate, reconstructed data. The COMFI operator used all actual locations within a 9x9 block centered on the interpolation bin (red square in Figure 4).

References