H033

A New Pseudo-acoustic Wave Equation for VTI Media

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SUMMARY

We propose a new approximate partial differential equation for qP-waves in transverse isotropy with a vertical symmetry axis (VTI) media. We analyse its relationship to two other published "pseudo-acoustic" VTI equations. All three pseudo-acoustic VTI wave equations are coupled systems of second-order PDEs in time, derived from the same dispersion relation for qP waves by introducing different auxiliary functions. The new method combines efficient implementation and low artifacts. Modeling and reverse-time migration are shown to validate the wave equation.
Introduction

Seismic anisotropy is observed in many exploration areas (e.g., the North Sea, offshore West Africa, the Canadian Foothills, and the Gulf of Mexico). Conventional isotropic migration methods for seismic imaging are insufficient in these areas. Many researchers have implemented two-way wave equation modeling and migration in anisotropic media with pseudo-acoustic approximations (Klie and Toro, 2001; Zhou et al., 2006; Hestholm, 2007). Alkhalifah (2000) introduced a pseudo-acoustic approximation for vertical transversely isotropic (VTI) media. Although this dispersion relation for a scalar wavefield has kinematics that are close to those of the qP-arrivals in the real elastic vector wavefield, it allows spurious SV-like events (Grechka et al., 2004). Based on Alkhalifah’s approximation, different space/time-domain wave equations have been proposed (Alkhalifah, 2000; Zhou et al., 2006). The following description focuses on a new pseudo-acoustic VTI wave equation also derived from Alkhalifah’s dispersion relation.

Method

Starting from the qP phase velocity equation in VTI media and setting the shear velocity along the symmetry axis to be zero, Alkhalifah (2000) heuristically obtains a dispersion relation for qP waves in 3D acoustic VTI media.

\[
\omega^4 - \left[ v_z^2 (k_x^2 + k_y^2) + v_z^2 k_z^2 \right] \omega^2 - v_z^2 (v_n^2 - v_z^2) (k_x^2 + k_y^2) k_z^2 = 0, \tag{1}
\]

where \( k_x, k_y, \) and \( k_z \) are wavenumbers in the \( x, y, \) and \( z \) directions; \( \omega \) is angular frequency; \( v_z \) is vertical qP-wave velocity; \( v_n = v_z \sqrt{1 + 2\delta} \) is the qP-wave normal moveout (NMO) velocity; \( v_z = v_z \sqrt{1 + 2\epsilon} \) is the horizontal qP velocity; and \( \epsilon \) and \( \delta \) are anisotropic parameters defined by Thomsen (1986). The equivalent partial differential equation (PDE) follows immediately as

\[
\frac{\partial^4 q}{\partial t^4} + v_z^2 \left( \frac{\partial^4 q}{\partial x^2 \partial t^2} + \frac{\partial^4 q}{\partial y^2 \partial t^2} \right) + v_z^2 \frac{\partial^4 q}{\partial z^2 \partial t^2} + v_z^2 (v_n^2 - v_z^2) \left( \frac{\partial^4 q}{\partial x^2 \partial z^2} + \frac{\partial^4 q}{\partial y^2 \partial z^2} \right) = 0, \tag{2}
\]

which is problematic to solve as it is a fourth-order equation in time.

The following three wave equations are all derived from equation (1). Each is a coupled system of second-order PDEs in time applicable when \( \epsilon - \delta \) is positive. This condition was shown by Grechka et al. (2004). The third wave equation is newly proposed here.

**Wave equation 1.** Alkhalifah (2000) derived the following coupled system of second-order PDEs for the wavefield function \( p(x, y, z, t) \), approximating the qP-wave, and the auxiliary wavefield function \( q(x, y, z, t) \):

\[
\frac{\partial^2 p}{\partial t^2} = v_z^2 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) + v_z^2 \frac{\partial^2 p}{\partial z^2} + v_z^2 (v_n^2 - v_z^2) \left( \frac{\partial^4 q}{\partial x^2 \partial z^2} + \frac{\partial^4 q}{\partial y^2 \partial z^2} \right), \tag{3a}
\]

\[
\frac{\partial^2 q}{\partial t^2} = p. \tag{3b}
\]
Wave equation 2. Zhou et al. (2006) used a different auxiliary function $q(x, y, z, t)$ to derive the following coupled system of equations for the wavefield function $p(x, y, z, t)$ approximating the qP-wave.

\[
\frac{\partial^2 p}{\partial t^2} = v_n^2 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 q}{\partial y^2} \right) + v_z^2 \frac{\partial^2 p}{\partial z^2}, \quad (4a)
\]

\[
\frac{\partial^2 q}{\partial t^2} = (v_n^2 - v_q^2) \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 q}{\partial y^2} \right). \quad (4b)
\]

Compared with equations (3a) and (3b), Zhou’s equations use lower-order derivatives and are more convenient for computational efficiency (Zhou et al., 2006).

Wave equation 3. Multiplying both sides of the original dispersion relation (1) with the wavefield $p(\omega, k_x, k_y, k_z)$, and introducing the new auxiliary function $\omega \left( v_n^2 - v_q^2 \right)(k_x^2 + k_y^2)$, equation (1) can be written as

\[
\omega^2 p(\omega, k_x, k_y, k_z) = v_n^2 (k_x^2 + k_y^2) p(\omega, k_x, k_y, k_z) + v_z^2 k_z^2 q(\omega, k_x, k_y, k_z).
\]

Applying an inverse Fourier transform to both sides of the previous two equations, using the correspondent relations: $i\omega \leftrightarrow \partial / \partial t, \ -ik_x \leftrightarrow \partial / \partial x, \ -ik_y \leftrightarrow \partial / \partial y, \ -ik_z \leftrightarrow \partial / \partial z$, and reorganising, the final pseudo-acoustic VTI equation can be written as

\[
\frac{\partial^2 p}{\partial t^2} = v_n^2 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) + v_z^2 \frac{\partial^2 p}{\partial z^2}, \quad (5a)
\]

\[
\frac{\partial^2 q}{\partial t^2} = v_n^2 \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) + v_z^2 \frac{\partial^2 q}{\partial z^2}. \quad (5b)
\]

For seismic forward modeling, we must inject the source function in the right side of both equations. The wavefield functions $p$ and $q$ have the same kinematics, which will be shown in the following analysis and discussion. The wave equation proposed in Zhou et al. (2006) can also be derived by a change of variables: $p = p' + q'$ and $q = p'$ in equations (5a) and (5b). Time stepping implemented with this new wave equation should be as efficient as the implementation of wave equation 2 given in Zhou et al. (2006), but does not require the extra memory storage of an intermediate variable $p+q$. The properties of the $p$ and $q$ wavefield functions of all three wave equations will be discussed in the following analysis.

Analysis and discussion

In a modeling experiment, time snapshots of wave propagation in a homogenous VTI anisotropic medium ($v_z = 3000$ m/s, $\varepsilon = 0.24$, and $\delta = 0.1$) are simulated using finite differencing. Figures 1(a) -1(f) correspond to the same time snapshot from modeling with each wave equation. The qP-wave wavefront and a diamond-shaped spurious SV-wave wavefront can clearly be seen. Figure 1(a) displays the $p$ wavefield of wave equation 1. The wavefronts of the qP-wave and spurious SV-wave are symmetric along the coordinate axes. Figure 1(b) displays the $p$ wavefield of wave equation 2 with similar wave phenomena. It appears that wave equation 2 introduces a stronger unwanted spurious SV-wave in the vertical direction. The $q$ wavefield of wave equation 2, displayed in Figure 1(c), is the same as the difference wavefield displayed in Figure 1(f). Figures 1(d) and 1(e) are the $p$ and $q$ wavefields of wave equation 3. The $p$ wavefield of wave equation 3, displayed in Figure 1(d), is quite
similar to the $p$ wavefield of wave equation 1, displayed in Figure 1(a). As expected, the $p$ wavefield from wave equation 2 is the same as the $q$ wavefield from wave equation 3.

Reverse-time migration was implemented with wave equation 3 using the $p$ wavefield. Figure 2 shows its impulse response in the same homogeneous anisotropic medium. An anisotropic salt model is chosen to verify this operator for complex media. Figure 3 displays the velocity, $\varepsilon$, and $\delta$ models. Prestack seismic data was generated from this model using elastic finite-difference modeling. Figure 4 displays the prestack anisotropic reverse-time migration image. The accurate imaging of the steep fault and salt flank can be clearly seen.

Figure 1: (a) Snapshot of $p$ wavefield with WE 1; (b) Snapshot of $p$ wavefield with WE 2; (c) Snapshot of $q$ wavefield with WE 2; (d) Snapshot of $p$ wavefield with WE 3; (e) Snapshot of $q$ wavefield with WE 3; (f) Difference of $p$ wavefield between WE 1 and WE 2.

Figure 2: Impulse response with $p$ wavefield function in wave equation 3.

Figure 3: Salt model, (a) velocity (ft/s); (b) epsilon; (c) delta.
Conclusions

We have proposed a new pseudo-acoustic VTI propagator, derived from the same dispersion relation as Alkhalifah (2000). Unsurprisingly, it provides the same kinematic approximation to the qP arrivals in the real elastic vector wavefield as those wave equations proposed by Alkhalifah (2000) and Zhou et al. (2006). Our modeling experiments of each coupled system of second-order PDEs also indicate that this new wave equation has a close dynamic match to Alkhalifah’s wave equation with a weaker unwanted spurious SV-wave than the Zhou et al. wave equation. An implementation of finite difference modeling with this new wave equation should be at least as computationally efficient as an implementation of the Zhou et al. wave equation. We have demonstrated this new wave equation for both pseudo-acoustic VTI modeling and migration.

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References


