Matching Pursuit Methods Applied to Multicomponent Marine Seismic Acquisition – The Issue of Crossline Aliasing

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SUMMARY

In this work we analyze the theoretical aspects of spatial aliasing in the crossline direction in marine seismic acquisition. We also explain the benefits of the additional measurements acquired by a multicomponent towed streamer, able to measure the three components of the particle velocity vector in addition to the pressure wavefield. We propose matching pursuit based techniques to reconstruct a 3D full bandwidth seismic wavefield on a fine receiver grid. The techniques that we describe process multicomponent seismic data; they calculate the desired 3D wavefield with satisfactory quality despite the severe aliasing that affects each of the individual input measurements in the crossline direction.
Introduction

In towed-streamer marine seismic acquisition, the crossline sampling can be irregular, and is typically coarse. Only a limited number of streamers can be towed, and to acquire data over a wider aperture, the streamer separation is often greater than may be desired. The recorded wavefield can be strongly aliased in the crossline direction. Consequently, most standard data-conditioning and processing techniques (e.g., deghosting, demultiple and velocity analysis) rely on approximations, such as assuming that seismic events are linear in the crossline direction or even that the crossline component of the angle of incidence is equal to zero (2D approximation).

Recently, Robertsson et al. (2008) introduced the concept of a multicomponent marine streamer, able to measure the full particle velocity vector in addition to the pressure wavefield; and showed that the value of a vector particle velocity measurement goes substantially beyond wavefield decomposition as it also helps to address the issue of crossline sampling. From physics, we know that the particle-velocity vector, $\vec{V}$, is related to the spatial gradient of the pressure wavefield, $P$: $\nabla P = -\rho \ddot{a} = -\rho \partial \vec{V} / \partial t$, where $\rho$ is the density of the medium. As pointed out by Robertsson et al. (2008), signal theory implies that the crossline component of the pressure gradient measured by a multicomponent streamer ($V_y$) allows interpolation in the crossline direction of the seismic pressure in a spatial bandwidth that is up to twice the Nyquist wavenumber (Linden, 1959). More recently, Amundsen et al. (2010) also showed how the same theory can be applied to ocean bottom node acquisition. Vassallo et al. (2010) introduced multichannel interpolation by matching pursuit (MIMAP) as a technique to combine the information of the pressure and the crossline component of particle velocity to achieve crossline reconstruction. This technique proved to have high dealiasing capabilities without relying on assumption such as the linearity of events in the crossline direction. Özbek et al. (2010) generalized the MIMAP technique to the joint-interpolation and 3D deghosting problem, proposing a technique called generalized matching pursuit (GMP) processing the pressure, $P$, the crossline and the vertical components of the particle velocity vector, $V_y$ and $V_z$. GMP can deghost the pressure wavefield in a 3D sense and reconstruct the upgoing and downgoing separated wavefields even when the input samples are affected by severe aliasing in the crossline direction. Compared to MIMAP, the GMP technique exploits the additional crossline information contained in the $V_z$ measurement to further increase its dealiasing potential.

In this work, we analyze how the aliased spectral replicas overlap in the frequency-wavenumber domain in the presence of coarse crossline sampling; we explain what the theory predicts to be recovered when gradients are available, and we introduce the increased dealiasing potential of MIMAP and GMP, showing its impact on synthetic examples.

Crossline aliasing and MIMAP

In marine acquisition, seismic waves obey the relation $k \leq 2\pi f / v$ in the frequency-wavenumber (f-k) domain, where $v$ denotes the apparent velocity: $v = c / \sin(\theta)$, with $c$ the speed of sound in water and $\theta$ the angle of incidence with respect to the vertical. Hence, if the minimum apparent velocity is known (e.g., 1500 m/s in water), the signal region in the f-k domain can be identified. This region is called the signal cone.

According to sampling theory, the combination of signal and its horizontal derivative allows the reconstruction of the wavefield that is subject to first-order aliasing, where first-order aliasing is the aliasing generated by the superposition of the base bandwidth with no more than one spectral replica.
resulting from spatial sampling. Figure 1 shows a schematic view of the areas of the f-k_y domain subject to aliasing in a marine acquisition when streamers are regularly spaced at 100m. The black bold lines represent the signal cone, assuming a minimum apparent velocity of 1500m/s. Sampling at 100m generates replicas of the signal cone in the k_y direction at every 0.01m\(^{-1}\). The replicas may overlap more than once (the intensity of the gray indicates the number of overlaps in the f-k regions), generating first-order and higher-order aliasing. As it is clear from Figure 1, provided that the pressure and its horizontal gradient are available, the multichannel sampling theorem would allow the reconstruction of the signal in f-k regions characterized by the two brightest gray shades (between -0.01 and 0.01m\(^{-1}\) and with a peak at f=22.5Hz, when k= -0.005 or 0.005 m\(^{-1}\)).

The MIMAP method (Vassallo et al., 2010) is a multicomponent reconstruction technique that models the signal as the superposition of a set of basis functions (i.e., complex exponentials). Each basis function is described by a set of parameters (i.e., amplitude, wavenumber and phase). The basis functions are selected iteratively to minimize a cost function evaluated as the residual at the input positions, simultaneously and optimally matching both the pressure signal and its horizontal gradient. Being completely data dependent, MIMAP has the capability to reconstruct data even in the presence of high order aliasing. To understand this phenomenon, let us consider a monochromatic wave, i.e., a sinusoid, sampled on a uniformly spaced grid. If another sinusoid has the same amplitude and phase as the original and its wavenumber differs from the first one by a multiple of the sampling rate, these two sinusoids will have exactly the same samples on the uniform grid. This is known as aliasing. When there is aliasing in single-component acquisition, it is not possible to identify the correct waveform from the acquired samples. On the other hand, if the two sinusoids have the same samples on a uniform grid, the gradients of these sinusoids cannot be the same on the regular grid. In fact, for a sinusoidal signal, the amplitude of the gradient is proportional to the amplitude and wavenumber of the signal. Hence, the samples of many sinusoids may fit to the data but among all those sinusoids, only the correct one will have a gradient that matches the gradient samples, as shown in Figure 2. This holds if there is no perfect spectral overlap of replicas of different events. The sinusoid example illustrates in a simple and intuitive way the potential of MIMAP to interpolate beyond twice the Nyquist rate. When interpolating in the crossline direction, the basis function that matches the data best, at every iteration, is the basis function that minimizes the joint residual on the pressure and the particle velocity inputs. When aliasing is present, thanks to the property mentioned above, the only spectral replica that simultaneously matches the two input measurements is the correct one. Figure 3 shows a simple example reproducing linear events with energy up to 65Hz and various incidence angles between ±90°, first decimated at 75m and then reconstructed using different techniques. It is clear how MIMAP (Figure 3g-h) improves on multichannel sinc, the interpolator recovering first order alias when spatial derivative is available in ideal conditions (Figure 3e-f).

Figure 2: Diagram showing that although two sinusoids at the sampling positions are identical (top) their gradients have different amplitudes (bottom).

Figure 3: simple synthetics in the t-y and f-k domains: (a, b) Input pressure, sampled at 75m; (c, d) sinc interpolated output; (e, f) output of multichannel sinc interpolator, using also the crossline gradients; (g, h) MIMAP output, using also the crossline gradients.
Joint interpolation and deghosting with matching pursuit and three-component signals

Özbek et al. (2010) showed that there is crossline information also in the vertical component of particle velocity when inverted through the use of the ghost model. For an intuitive explanation as to why this should be the case, consider an upgoing plane wave propagating purely in the crossline direction as shown in Figure 4. The wavefield is recorded at the streamers represented by the dark colored triangles, before being reflected downward at the free surface. The downgoing wavefield recorded at the streamers has actually passed through different crossline spatial locations (light colored triangles) compared to the upgoing wavefield. The locations of these “virtual” crossline sampling points depend on the angle of incidence; hence, the downgoing wavefield is a replica of the upgoing wavefield, where each angle-dependent time delay corresponds to an angle-dependent crossline spatial delay. In practice, the downgoing wavefield carries to the receivers the information related to the upgoing wavefield at different crossline positions. As $V_z$ carries complementary ghost information with respect to the pressure, this component can be used, in combination with $P$ and $V_y$, to extract the crossline information that is implicitly associated to the ghost reflection. We can produce a linear system modeling the three seismic measurements, $P$, $V_y$ and $V_z$ as filtered versions of the upgoing pressure, $P_{up}$. In this system, the forward filters are given by the different ghost models acting on the different components in the crossline direction:

$$
\begin{align*}
P(\mathbf{\bar{f}}, \mathbf{\bar{k}}, z) &= \left[1 - \exp(2j\mathbf{k} \cdot (\mathbf{\bar{f}}, \mathbf{\bar{k}}, z))\right] P_{up}(\mathbf{\bar{f}}, \mathbf{\bar{k}}, z) \triangleq H_1(k_x) P_{up}(k_x) \\
V_y(\mathbf{\bar{f}}, \mathbf{\bar{k}}, z) &= \frac{k_y}{2\pi f} \left[1 - \exp(2j\mathbf{k} \cdot (\mathbf{\bar{f}}, \mathbf{\bar{k}}, z))\right] P_{up} \triangleq H_2(k_y) P_{up}(k_y) \\
V_z(\mathbf{\bar{f}}, \mathbf{\bar{k}}, z) &= \frac{k_z}{2\pi f} \left[1 + \exp(2j\mathbf{k} \cdot (\mathbf{\bar{f}}, \mathbf{\bar{k}}, z))\right] P_{up} \triangleq H_3(k_z) P_{up}(k_z)
\end{align*}
$$

where $f$ represents the temporal frequency, $k_x$, $k_y$ and $k_z$, the inline, crossline and vertical wavenumbers, respectively, and $Z$ the depth of the receiver. In theory, equation (1) reproduces the scheme of the generalized sampling expansion (Papoulis, 1977) and this suggests that the inversion of equation (1) can enable the reconstruction of the upgoing pressure wavefield by processing the three input components, $P$, $V_y$ and $V_z$, affected by up to the second order alias. Generalized matching pursuit (GMP) is a technique that solves the problem modeled above by making use of matching pursuit. It can be seen as the generalization of MIMAP. In the case of GMP, the basis functions are meant to describe the 3D upgoing wavefield and the forward linear filters $H_m(k)$ from equation (1) are applied to them. The filtered basis functions are then iteratively matched to the multicomponent measurements. The basis functions that, once forward filtered, jointly best match all the input signals are used to reconstruct the desired output. As a data-dependent iterative method based on matching pursuit, GMP can achieve simultaneous 3D up-down wavefield separation and crossline reconstruction in realistic conditions, with finite and irregular samples. It also extends the dealiasing potential of MIMAP to the three-component case, increasing its robustness to noise and its resolution.

The dataset used for our tests was created by finite-difference modeling, simulating a 3D multicomponent survey over a complex geological environment. The source signature spectrum is flat up to 30Hz. The acquisition geometry consists of a regularly spaced receiver carpet located at 50m of depth: the unusual depth was chosen to place the pressure ghost notch within the 30Hz bandwidth. Figure 5, left, shows the FKxKy transform of the total (up- and down-going) pressure wavefield of a selected shot gather, sampled over a 25x25m spatial grid. The ghost notch is recognizable by the lack of energy in the low wavenumbers in the 15Hz slice, and a circularly shaped notch in the 20Hz and 25Hz slices. The events that are not affected by the notch are still affected by the constructive interference of the ghost. The transform of the total pressure wavefield after decimation of the data to
150m in the crossline direction is shown in Figure 5, centre. First order aliasing starts just above 5Hz, and the order of aliasing grows significantly with frequency. Figure 5, right, shows the FKxKy transform of the upgoing (deghosted) pressure wavefield reconstructed to 25x25m by GMP. Together with a very significant dealiasing action, the ghost notch has been filled and the resulting energy distribution is now more uniform. Repeating these experiments with the addition of realistic noise levels to the input gradients has also demonstrated significant robustness to noise in GMP.

**Figure 5:** Seismic synthetics in FKxKy domain. From left to right: total pressure wavefield, sampled over a 25x25m grid; total pressure wavefield at 150m crossline spacing; upgoing pressure wavefield, sampled over a 25x25m grid generated using GMP, processing P, Vy and Vz at 150m in crossline.

**Conclusions**

We have shown how the matching pursuit techniques can be successfully applied to multicomponent data and how the data dependency of these methods enables high-order dealiasing without relying on assumptions such as the linearity of the events. We proposed MIMAP and GMP for the problems of crossline interpolation and joint interpolation and deghosting for data acquired by a multicomponent marine streamer that measures the full particle velocity vector in addition to the pressure.

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**References**


