BGP11

Elastic gaussian beam imaging of walk-away VSP data.

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SUMMARY

The approach to true amplitude seismic imaging for walk-away Vertical Seismic Profile (VSP) multi component data is presented and discussed. This approach is migration procedure based on weighted summation of prestack data. True amplitude weights are computed with application of Gaussian beams (GB). The couple of beams is used in order to compute true amplitude selective image of the rapid velocity variation. The total true amplitude image is constructed by superposition of selective ones being computed for a range of available dip angles.

Shooting from the bottom overcomes difficulties due to multi arrivals of seismic energy which are common for complicated velocity models (in particular salt intrusions). In addition, the global regularity of Gaussian beams permits one not take care of, is the ray field regular or not. The use of P- and S-wave Gaussian beams provides the possibility to handle raw multi component data without of their preliminary separation to PP and PS waves. The true amplitude selective images computed for a variety of opening angles gives possibility to implement AVO analysis.
Introduction

VSP migration is essentially a pre-stack migration technique and is based on the following imaging condition: location of reflector/scatterer is assigned to positions where backward extrapolated wave field is in the same phase as downward extrapolated source wave field.

There are a lot of ways for its implementation producing variety of VSP imaging procedures (see, for example, (Fei and Liu, 2006)). But so far there is no true-amplitude version of these procedures besides ones based on presented in (Miller et al., 1987) asymptotic inversion of Generalized Radon Transform (GRT). This approach essentially uses ray-based representation for wave fields and meets troubles if ray field is not regular for some source/receiver positions.

The techniques presented below provides the true amplitude image of pre-stack VSP data originated from the approach proposed and developed in (Tcheverda and Protasov, 2006) for surface multi-shot/multi-offset single component data. It provides the user with an image of the subsurface in the vicinity of a borehole and is implemented by extrapolation of wave fields from acquisition system towards some fixed imaging point along specified Gaussian beams.

The possibility to shoot P- and S-wave Gaussian beams results in AVO-like analysis of selective images computed for a set of opening angles. Another feature of elastic imaging algorithm comes from its multicomponent nature. Elastic imaging algorithm eliminates artifacts produced by P to S conversion and therefore does not need in preliminary separation of the data to P and S waves.

Theory and method

Let us consider half-plane $\mathbb{R}^2_+ = \{ x, z : z > 0 \}$, filled with heterogeneous elastic media with Lamé’s parameters and density:

$$\lambda = \lambda_0 + \lambda_1, \quad \mu = \mu_0 + \mu_1, \quad \rho = \rho_0 + \rho_1.$$ Parameters $\lambda_0(x,z), \mu_0(x,z), \rho_0(x,z)$ introduce a priori known reference medium (background), while $\lambda_1(x,z), \mu_1(x,z), \rho_1(x,z)$ are responsible for its fast variations below referred to as reflectivity. Let us suppose that along axis $x=0$ (treated as a “borehole”) the wave field scattered/reflected by reflectivity is known for a range of source positions:

$$\eta \leq z < \eta + H,$$

where $\eta(x,z)$ is Green’s matrix computed for background. The problem is to recover functions $\lambda_1, \mu_1, \rho_1$ or their combinations by the data (1).

In order to reconstruct “reflectivity” at the point $(x,z)$ let us trace a couple of P-rays towards acquisition system and connect with them a couple of Gaussian beams (see Fig.1). Next, let us compute normal derivatives of each Gaussian beam together with their potentials at the source positions:

$$L_1 < \tilde{u}_0 > = \frac{\partial}{\partial x}(\lambda \cdot \text{div} \tilde{u}_0 + 2 \mu \frac{\partial u_0}{\partial x} + \frac{\partial u_0}{\partial z}) + \frac{\partial}{\partial z}(\mu \cdot (\frac{\partial u_0}{\partial x} + \frac{\partial u_0}{\partial z}) + \lambda \frac{\partial u_0}{\partial x}),$$

while $\tilde{u}_0(x,y,z,\eta,\omega)$ is the incident wave field which propagates within the reference medium from the source point $(x_0, \eta, 0, \omega)$ and taken at some current point $(x, \eta, z)$ within the target area. In its own turn $\Gamma(0, z; \xi, \eta, \omega)$ is Green’s matrix computed for background. The problem is to recover functions $\lambda_1, \mu_1, \rho_1$ or their combinations by the data (1).

Use of the incident wave in terms of potential and application of Green’s theorem followed by asymptotic analysis on the base of stationary phase theorem gives the following relation:

$$\int\int T_{gb}^g(x_0, \alpha, \beta, \omega) \cdot \tilde{T}_{gb}^g(z_0, \alpha, \beta, \omega) dx_0 dz_0 d\alpha d\omega = \int\int f_{\beta}(y) \cdot \exp(i \cdot \beta \cdot (x - y)) dy.$$

Borehole Geophysics Workshop - Emphasis on 3D VSP
16-19 January 2011, Istanbul, Turkey
With function
\[
f_\beta = \frac{\lambda_1 + 2\mu_1 \cos^2(2\beta) + \rho_0^2 v_0^2 \rho_1 \cos(2\beta)}{4\rho_0 v_0^2 \cos^2(\beta)}. \tag{3}
\]
As one can see, the right hand side of (2) is nothing else but superposition of 2D spatial Fourier transform applied to desired function \(f_\beta\) followed by its quasiinverse. It is not the exact inversion of Fourier transform, because it is performed not over complete phase space, but over its subdomain \(X_{par}\) only. This subdomain is circular sector which is defined by frequency bandwidth of source function \((\omega_1, \omega_2)\) and available range of dip angles \((\alpha_1, \alpha_2)\) (see Fig.2):
\[
\left\{ (p_x, p_z) : \alpha_1 \leq \frac{v_0^2(p_x^2 + p_z^2)}{2\cos\beta} \leq \omega_2; \alpha_1 \leq -\arctan \frac{p_x}{p_z} \leq \alpha_2 \right\}, \tag{4}
\]
Particularly, this operator will not change function which has support of its Fourier spectrum entirely within the set \(X_{par}\). And vice versa, if this support is completely out of \(X_{par}\), it belongs to the kernel of this operator and will be not imaged by the procedure at all.

**Figure 1** Geometry of GB imaging.  
**Figure 2** Set of partial reconstruction.

**Examples**

Numerical experiments were done for finite-difference synthetic data generated for well known velocity model Sigsbee2A. The geometry of acquisition system is presented in the Fig.3 (red triangles). It is chosen in order to recover “reflectivity” within target area marked by the black rectangle. Multicomponent synthetic data are computed by second order finite-difference staggered grid schemes (Virieux). In the Fig.4 VSP x- and z-components seismograms are presented for some source position. In order to compute true-amplitude image the range of dip angles \([\alpha_1, \alpha_2] = [-20^\circ : 5^\circ : 5^\circ]\) is used. In the Fig. 5 results of 2C GB imaging (top) for opening angles \(\beta=35^\circ\) and \(\beta=40^\circ\) are presented in comparison with the same function computed for true full model (bottom). As one can see in both images “width of the interfaces” becomes wider when opening angle \(\beta\) is increased. It is due to change of variables in spectral domain. Also one can follow influence of the illumination conditions on the 2C GB imaging results. We can see in the case of walk-away VSP data illumination conditions of different regions can be rather different. Because illumination is in our hands therefore still we can choose some regions and some opening angles \(\beta\) for the regions with more or less uniform illumination and provide their recovery of elastic properties. For example we take one image trace on \(x = 1800m\) and range of \(\beta = [30^\circ : 1^\circ : 60^\circ]\) and try to recover some elastic parameters. One can see...
this image trace with respect to opening angle $\beta$ in the Fig.6 (top). One can see very good correlation of the recovered and true amplitudes. We did inversion with respect to Lamé’s parameters - $\lambda_1$, $\mu_1$ and normalized density $\rho_1(v_{p1}^0)^2$. The result of inversion one can find in the Fig.6 (bottom). Recovery of parameter $\lambda_1$ is almost ideal. But correlation of recovered and true traces for other 2 parameters is poor while recovery of amplitudes is rather good.

**Figure 3** Gulfaks model. Acquisition system and target area.

**Figure 4** VSP seismograms. x-component (top), z-component (bottom).

**Figure 5** 2c GB imaging (top) and true images (bottom) for different opening angles.

**Figure 6** 2c GB elastic image traces (blue), true image traces (red). Reflection coefficient (top) and elastic parameters (bottom).
Conclusions

It should be noted that proposed imaging procedure provides a set of “selective images”. Their main features are as follows:

- if spatial spectrum of rapid perturbation of the reference medium completely belongs to the set of partial reconstruction it is recovered with true amplitude;
- if the spatial spectrum of a local object lies entirely outside of the set of partial reconstruction, this object is completely “invisible”;
- if spatial spectrum of a local object possesses nonempty intersection with a set of partial reconstruction selective image will be made from orthogonal projection of desired perturbation onto the set of partial reconstruction.

On this base one can conclude, that any singular object like diffractor/scatterer, crack, fault, pinch and so on possesses extended spatial spectrum and, so, will be presented for a wide range of selective images. On the contrast, any regular interface possesses very narrow spatial spectrum and, so, one can easy choose geometry of the Gaussian beams providing the set of partial reconstruction without of this spectrum. This opens a possibility to get reliable image of low contrast singular objects of subseismic scale, like cavernous, cracks and fractures.

The key to the success of Gaussian beam based true-amplitude migration of VSP data is a good quality of the migration (reference) model. It should guarantee the correct travel times between sufficiently distant from each other points and contain rather smooth interfaces.

It is the geometry of the acquisition system and structure of the reference medium determines the set of partial reconstruction. The method implements the true-amplitude imaging of 2C VSP data. As one can see, the recovered function $f_\beta$ is exactly linearized reflection coefficient. Increasing of the opening angle $\beta$ allows implementation of AVO analysis and recovery of elastic properties of the medium below the interface. It is worth mentioning that this approach does not need in any preliminary separation of P- and S-waves.

Acknowledgments

The research described in this publication was done in cooperation with Moscow Schlumberger Research and partially supported by RFBR, grants 08-05-00265, 10-05-00233. Authors are grateful to Dmitri M. Vishnevsky for the code for elastic finite difference simulation.

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