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Wavepath-consistent Effective Q Estimation for Q-compensated Reverse-time Migration

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SUMMARY

We propose a method for compensating for Q effects in reverse-time migration. The new method relies upon running acoustic propagators twice to estimate attenuated traveltimes along wavepaths that are then used to filter the conventional source and receiver wavefields to compensate for amplitude and phase effects prior to imaging. As this method does not rely upon two-way time-domain viscoacoustic propagators, we avoid the difficulty of stabilizing backward propagation of the receiver wavefield when amplitudes are amplified. By separately applying phase and amplitude filters post modelling we can efficiently tune the stabilization of the amplitude amplification filter without altering the phase compensation. If required for model building, we can produce a suite of images using different percentages of the original Q model at little extra cost.
Introduction

Amplitude attenuation and wavelet distortion have been observed on seismic data due to anelastic properties of the earth. For example, strong attenuation of seismic P-waves can result from gas trapped in overburden structures. This results in a dimming of migrated amplitudes below the gas anomaly as well as reduced resolution in the image caused by high-frequency energy loss and phase distortion. Correcting for these effects in seismic processing will make the final image more interpretable.

Compensation for seismic attenuation was first performed in the unmigrated domain by an inverse Q filter (e.g. Bickel and Natarajan, 1985). Based on a one-dimensional backward propagation, these methods cannot handle real geological complexity. Because anelastic attenuation and dispersion occur during wave propagation, it is natural to consider correcting them in a depth migration. However, migration methods usually treat the earth model as a lossless acoustic medium, and only correct for the amplitude effects of geometric spreading. As one-way wave-equation migrations are formulated in the frequency domain, it is straightforward to take care of frequency-dependent dissipation (Yu et al. 2002). Reverse-time migration (RTM) based on directly solving the two-way wave equation provides a superior way to image complex geologic regions. It is also straightforward to incorporate an attenuation correction into a frequency-domain implementation of RTM (e.g. Causse and Ursin, 2000). However, to incorporate an attenuation correction into a conventional implementation of RTM, we must formulate a time-domain wave equation to model the viscoacoustic effects, which is not a straightforward task. Additionally, the backward propagation of the receiver wavefield in a viscoacoustic RTM must model amplification (not amplitude attenuation), which can be unstable.

Carcione et al. (1988) designed a system of equations of motion to perform viscoacoustic modelling and introduced memory variables to eliminate storing the entire strain history required by the time convolution. Carcione (2010) generalized the Fourier pseudospectral method to the case of derivatives of nonnatural order (fractional derivatives) and irrational powers of the differential operators. To model spatially constant Q propagation the approach is highly efficient because it does not require memory variables or additional spatial derivatives. However, to model spatially variable Q, wavefields would have to be stepped for a selection of reference Q values and then interpolated to the actual Q value at each spatial location. Both these approaches model the combined effect of amplitude attenuation and velocity dispersion, and so it is likely that any regularization employed to stabilize the backward propagation of the receiver wavefield will affect the phase as well as damp the amplification.

There have been few attempts at implementing Q-compensated time-domain RTM. Deng and McMechan (2007) proposed implementing RTM using a modified scalar acoustic wave equation that only compensated for amplitude attenuation effects and ignored phase effects. Zhang et al. (2010) derived a pseudodifferential equation to model viscoacoustic waves based on Kjartansson’s (1979) dispersion relation, and applied it to RTM with regularization for the backward propagation of the receiver wavefield. Their pseudodifferential equation does involve separate operators for phase and amplitude effects, but it is not clear how they go about performing regularization or the cost of running such a propagator.

We chose to not attempt to derive an efficient time-domain viscoacoustic propagator that can be applied within RTM in a stable fashion. Rather, we propose a new method that relies upon running standard acoustic propagators twice to estimate attenuated traveltimes along wavepaths. These attenuated traveltimes are then used to filter the conventional source and receiver wavefields to compensate for amplitude and phase effects prior to imaging. This is somewhat analogous to how Q effects can be incorporated into Kirchhoff depth migrations, but with wavepath (rather than raypath) consistent attenuated travelt ime estimates.

Method
The effect of intrinsic attenuation on a wave propagating through a viscoacoustic medium can be characterized in terms of the attenuated traveltime, \( t^* \), defined by

\[
    t^* = \int_{path} v^{-1}(s)Q^{-1}(s)ds = \int_{path} Q^{-1}(t)dt. \tag{1}
\]

The attenuated traveltime integrates the effects of velocity, \( v \), and \( Q \) along the propagation path. With ray-based modelling, the attenuated traveltime can be easily computed by reintegrating \( Q \) along a traced ray. For two-way wavefield extrapolation modelling we do not have such flexibility as we do not explicitly calculate wavefields along individual wavepaths. However, an estimate of attenuated traveltime can be computed for the full wavefield (at all points in space, \( x \), and time, \( t \)) by running two modelling experiments (either in parallel or sequentially).

For ease, we discuss modelling using second-order in time finite-difference modelling of the scalar acoustic wave equation. In practice the approach will work with any acoustic modelling algorithm that steps the wavefield in time, including anisotropic acoustic modelling using coupled equations (e.g. Fletcher et al., 2009) or any recursive integral time extrapolation algorithm (Fowler et al., 2010). The first experiment is normal modelling, stepping the wavefield as:

\[
P(x; t + dt) = 2P(x; t) - P(x; t - dt) + dt^2 v(x)^2 \nabla^2 P(x; t), \tag{2}
\]

where \( dt \) is the time step. The second modelling experiment (written here as pseudocode) additionally integrates the effect of \( Q \) over time as follows:

\[
P_0(x; t + dt) = 2P_0(x; t) - P_0(x; t - dt) + dt^2 v(x)^2 \nabla^2 P_0(x; t)
\]

where \( \alpha \) is a constant scalar. Now the following transformation of the two wavefields \( P \) and \( P_0 \) yields an estimate of \( t^* \) at a point in space and time that the wavefield has experienced

\[
t^*(x; t) = \ln(P(x; t)/P_0(x; t)) / \alpha. \tag{4}
\]

In practice, this approach will not completely handle multipathing in the wavefield correctly. If there are two arrivals at a single point in space and time, then the attenuated traveltime will be erroneous. It may also be necessary to ensure that the \( Q \) model is sufficiently smooth so as not to affect the kinematics of the wavefield \( P_0 \). The attenuated traveltimes calculated in equation (4) can then be used to design Q-modelling/compensation filters of the form

\[
    F(t^*, \omega) = \exp \left[ \mp \frac{\omega t^*}{2} \pm i\frac{\omega t^*}{\pi} \ln \left( \frac{\omega}{\omega_0} \right) \right],
\]

where \( \omega \) is angular frequency and \( \omega_0 \) is a reference angular frequency. These filters can be applied to the wavefield, \( P \), from the modelling in equation (2) to simulate modelling/compensation of \( Q \) effects. We apply these filters using the filter bank method proposed by Ferber (2005). Using the above method to simulate viscoacoustic propagation, we propose the following prestack RTM algorithm to compensate for the absorption effects in the image.

### Q-compensated RTM algorithm (Q-RTM)

1. **Source side**
   a. Forward propagate (equation (2)) to obtain wavefield \( S(x; t) \).
   b. Forward propagate, integrating \( Q \) (equation (3)) to obtain wavefield \( S_0(x; t) \).
   c. Calculate \( t^*_0(x; t) \) (equation(4)) from \( S(x; t) \) and \( S_0(x; t) \).
   d. Apply modelling filters (equation (5)) to \( S(x; t) \).
2. **Receiver side**
   a. Backward propagate (equation (2)) to obtain wavefield \( R(x; t) \).
   b. Backward propagate, integrating the \( Q \) (equation (3)) to obtain wavefield \( R_0(x; t) \).
   c. Calculate \( t^*_0(x; t) \) (equation(4)) from \( R(x; t) \) and \( R_0(x; t) \).
   d. Apply compensation filters (equation (5)) to \( R(x; t) \).
3. **Form image from \( S(x; t) \) and \( R(x; t) \).**
Without steps 1(b)-1(d) and 2(b)-2(d), this is just standard RTM. Including these steps clearly doubles the number of wavefields to propagate. We do not go into detail here, but note that these steps require access to the source and receiver wavefields ordered with time as the fast dimension. The standard imaging condition for acoustic prestack RTM employs a zero-lag crosscorrelation, which can be implemented without sorting the source and receiver wavefields to have time as the fast dimension. However, implementation of a more sophisticated deconvolution imaging condition would also require the source and receiver wavefield to be ordered with time as the fast dimension.

Based on a raytracing tomography algorithm, Cavalca et al. (2011) proposed a generalized inversion method to estimate the attenuation loss and to obtain a model for $Q$. If we were to scale the $Q$ model by a constant factor, $k$, then the effect on $t^*$ would be to scale by a constant $1/k$. Therefore, at little extra computation cost, it is possible to produce multiple RTM images using scaled versions of the $Q$ model. This could have potential use for $Q$ model building (particularly in areas where conventional ray-based $Q$ tomography fails).

**Examples**

We test this method using a viscoacoustic Gullfaks model. Figure 1 displays a velocity based on North Sea geology and an interval $Q$ model: a very low ($Q=20$) anomaly embedded in an attenuating background ($Q=120$) below the water layer ($Q=5000$). Prestack synthetic data were generated using the viscoacoustic finite-difference algorithm of Robertson et al. (1994). One hundred and fifty shot gathers were modelled. Figure 2 displays results of applying prestack RTM using the unsmoothed models in Figure 1. The effect of the $Q$ anomaly can be seen with acoustic RTM in a dimming of amplitudes along reflectors underneath the anomaly as well as clear evidence of phase distortions (most notably in the sag on the base reflector). The result from applying the Q-RTM algorithm described above is plotted on the same scale. The water bottom (which is not affected by $Q$) is consistent between both. Reflectors beneath the water layer can be seen to differ. As the background $Q$ represents fairly strong attenuation, the Q-RTM amplitudes are noticeably compensated (amplitude and phase) everywhere. The effect of the very low ($Q=20$) anomaly is less visible in the image, with less dimming of amplitudes along reflectors underneath the anomaly and more consistent phase.

**Conclusions**

We have proposed a new approach to incorporating $Q$ effects into prestack RTM. By separately applying phase and amplitude filters post modelling to the source and receiver wavefield before imaging, we can efficiently tune the stabilization of the amplitude amplification filter without altering the phase compensation. If required, for model building, we can produce a suite of images using different percentages of the original $Q$ model at little extra cost.

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**References**


Figure 1 (left) velocity. (right) Q (Q=20 anomaly embedded in a Q=120 background).

Figure 2 RTM with a global source illumination normalisation. (left) standard, (right) Q-compensated.