Estimation of the formation shear and borehole fluid slownesses using sonic dispersion data in the presence of a drill collar

SUMMARY

Compressional and shear slownesses are widely used in the mechanical characterization and lithological interpretation of formations in the oil and gas industry. A traditional way to estimate the formation shear slowness consists of minimizing differences between a measured and model-based sonic dispersions. However, this procedure requires repeated computations of model-based dispersions or a look-up table of dispersion data. A pre-computed look-up table of dispersion data as a function of system parameters may also have to be interpolated that can introduce additional inversion error. Either of these procedures results in expensive computational costs. To overcome these limitations, we introduce a parametric inversion methodology where the cost function is derived from the corresponding waveguide dispersion model. Hence, the exact solution can be always obtained with noise-free data. Inversion efficiency is also improved by avoiding an explicit computation of wideband dispersion curves. Since the borehole fluid slowness is usually not correctly known, the inversion algorithm has been extended to simultaneously estimate the formation shear and borehole fluid slownesses. We demonstrate validity of the proposed inversion algorithm using synthetic data for modal dispersions for a fluid-filled borehole in the presence of a drill-collars in both the fast and slow formations. Excellent agreement between the input modal dispersion and that computed using the inverted shear slowness confirms the accuracy of the inversion algorithm.

INTRODUCTION

Sonic logging-while-drilling (LWD) is carried out in the presence of a drill collar (Figure 1). A concentrically placed drill-collars in a fluid-filled borehole behaves like a strong waveguide and can cause significant perturbations to the borehole Stoneley, flexural and quadrupole dispersions. Therefore, it is necessary to account for the presence of a drill collar in the forward modeling of borehole dispersions. A model-based inversion of any of the borehole dispersions for estimating the formation shear slowness minimizes differences between the measured and modeled dispersions by varying the shear slowness for a given set of other system parameters. The other set of parameters include the borehole diameter (estimated from the drill bit size); mud density (based on the mud composition); drill-collar inner and outer diameters, drill-collar mass density and material elastic properties; formation mass density and compressional slowness (estimated from the compressional headwave logging).

This paper describes a new workflow for estimating the formation shear and borehole fluid slownesses from the inversion of a band-limited borehole Stoneley, flexural or quadrupole dispersion. This workflow is based on a parametric inversion algorithm described by (Braunisch et al., 2000; Braunisch, 2001) for estimating the formation shear slowness in an open-hole environment and in the absence of any sonic tool structure in the borehole fluid. In addition, we analyze sensitivity of the borehole Stoneley, flexural and quadrupole dispersions to various system parameters. Such sensitivity analyses are particularly useful in an optimal bandwidth selection for the inversion of borehole dispersions for the unknown formation shear slowness.

Figure 1: A concentric drill collar in a fluid-filled borehole.

THEORY

Modal dispersions of a fluid-filled borehole in the presence of a drill collar

We derive the existence condition for a borehole guided mode using a fundamental solution for the displacement and stresses associated with the elastic wave propagation in cylindrical structures. The existence of guided borehole modes in the presence of a drill collar can be expressed in terms of the roots of the boundary condition determinant (Hsu and Sinha, 1998; Sinha et al., 2009). This can be written as

$$D(k_z, \omega, \bar{x}) = 0,$$

where $D$ is the determinant of the system matrix (it is a 12 by 12 matrix for the model considered), $k_z$ is the wavenumber in the direction of propagation, $\omega$ is the angular frequency, and $\bar{x}$ is the vector that contains the geometrical and material parameters of the model, such as the formation shear slowness etc..

When the parameter vector $\bar{x}$ is given in the model, solutions to equation (1) are smooth curves in the $(k_z - \omega)$ plane, written as $k_z(\omega, \bar{x})$. The modal dispersions are then obtained in the slowness-frequency domain by defining the phase slowness as $Re\{k_z\}/\omega$. Numerically, these dispersions can be calculated by finding roots of $k_z$ in equation (1) along a smooth curve in the $\omega$ domain. This can be implemented using the complex Newton-Raphson method (Press et al., 1992).
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Parametric inversion of modal dispersion data

To solve the inverse problem, the objective is to estimate certain unknown components of $\vec{x}$ from a bandlimited, possibly noisy samples of one or more modal dispersion data. Following the work in (Braunisch et al., 2000; Braunisch, 2001), we choose a so-called “guidance mismatch” as our cost function. Mathematically speaking, suppose we have $M$ measured pairs $(\omega_i, k_{ij})$ that satisfy

$$k_{ij} = k_i(\omega_i, \vec{x}) + n_i, i = 1, 2, ..., M,$$

(2)

where $n_i$ is the noise in the data, we want to find $N$ unknown components of $\vec{x}$ that minimize the following cost function,

$$\|\vec{x}(\vec{x})\|^2 = \sum_{i=1}^{M} |D(k_{ij}, \omega_i, \vec{x})|^2.$$

(3)

Usually $N$ is less than the dimension of $\vec{x}$, since we assume some of its components can be obtained from other logging measurements. Typically in this paper, our primary interest is to estimate the formation shear and borehole fluid slownesses. Therefore, $N = 1$ when we estimate formation shear slowness alone, and $N = 2$ when we estimate both of them simultaneously.

Notice that, with noise-free data, i.e., $n_i = 0$ for $i = 1, 2, ..., M$, the cost function in equation (3) can be made zero. This suggests the cost function is well defined. On the other hand, we could always assume $N \leq M$, which provides sufficient data to find the unknowns. Numerically, this nonlinear least square problem can be solved by the Gauss-Newton method (Gill et al., 1981), where the partial derivatives in the Jacobian are computed using the finite-difference method.

### COMPUTATIONAL RESULTS

Testing examples of modal dispersion curves

![Figure 2: Fast formation: (a) Synthetic waveforms generated by a monopole source; (b) Synthetic waveforms generated by a dipole source; (c) Synthetic waveforms generated by a quadrupole source; (d) Modal dispersions obtained by processing synthetic waveforms by a modified matrix pencil algorithm (circles) and a mode-search algorithm (solid lines).](image)

![Figure 3: Slow formation: (a) Synthetic waveforms generated by a monopole source; (b) Synthetic waveforms generated by a dipole source; (c) Synthetic waveforms generated by a quadrupole source; (d) Modal dispersions obtained by processing synthetic waveforms by a modified matrix pencil algorithm (circles) and a mode-search algorithm (solid lines).](image)

Table 1: Material parameters for cylindrical components

<table>
<thead>
<tr>
<th>Material</th>
<th>$v_p$ (m/s)</th>
<th>$v_s$ (m/s)</th>
<th>$\rho$ (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast formation</td>
<td>3658</td>
<td>2032</td>
<td>2350</td>
</tr>
<tr>
<td>Slow formation</td>
<td>2478</td>
<td>1016</td>
<td>2270</td>
</tr>
<tr>
<td>Drill collar</td>
<td>5650</td>
<td>3170</td>
<td>7830</td>
</tr>
<tr>
<td>Borehole fluid</td>
<td>1500</td>
<td>–</td>
<td>1000</td>
</tr>
</tbody>
</table>

Table 1: Material parameters for cylindrical components

To check the consistency of our mode-search formulation and implementation, we first use the dyadic Green’s function methodology (Lu and Liu, 1994) to generate the borehole dispersions for the same geometry. This dyadic Green’s function methodology produces synthetic waveforms at an array of receivers generated by a monopole, dipole, or quadrupole source placed on the borehole axis. The synthetic waveforms are then processed by a modified matrix pencil algorithm to isolate the non-dispersive and dispersive arrivals in the wavetrain (Lang et al., 1987). Synthetic waveforms have been recorded at 51 receivers placed in the annulus between the drill collar and the formations with an inter-receiver spacing of 10.16 cm (4 inches), where the distance between the transmitter and the first receiver is 0.9144 m (3 ft). Figures 2 and 3 show synthetic waveforms for a monopole, dipole and quadrupole source for a
fast and slow formations (see Table 1). In Figures 2(d) and 3(d),
the modal dispersions calculated by our mode-search algo-
rithm are also overlapped to confirm its agreement with the
other formulation.

**Sensitivity of borehole dispersions to small perturbations
in model parameters**

Before we solve the inverse problem, it is important to deter-
mine frequency bands that we should use to invert for a par-
ticular system parameter. Obviously, we should use dispersion
data in a frequency band that is most sensitive to the param-
ters to be inverted for (Sinha, 1997). Therefore, we compute
the sensitivity function for a given parameter $p$ in the model
defined by the following equation

$$
\text{Sensitivity}(f, p) = \frac{p}{S(f)} \frac{\partial S(f)}{\partial p},
$$

where $S(f)$ denotes the slowness as a function of frequency $f$.
Numerically, the partial differential is calculated by a finite-
difference method using a forward step equal to $0.01\% \times p$.
Figure 4 illustrates the sensitivity functions of the Stoneley and
quadrupole dispersion curves for the fast formation.

![Figure 4: Sensitivity of the Stoneley (a) and quadrupole (b)
dispersions to small perturbations in model parameters for a
fast formation.](image)

**Estimation of formation shear slowness and borehole fluid
slowness**

Next we apply the aforementioned parametric inversion algo-
rithm to estimate the formation shear slowness from modal dis-
persions of a fluid-filled borehole in the presence of a drill col-
lar. The modal dispersions for a fast formation are shown in
Figure 2(d).

![Figure 5: (a) Relative error in the inverted shear slowness as a
function of uniform shift (or error) in the input Stoneley dis-
persion; (b) Relative errors in the inverted formation shear and
mud slownesses as a function of uniform shift (or error) in the
input Stoneley dispersion.](image)

Since the Stoneley dispersion in the frequency band from $1 - 3$
kHz is more energetic and less influenced by the near-wellbore
alteration, we use 9 samples (slowness-frequency pairs) within
this band with a fixed sampling step of $0.25\ kHz$. We first
invert for the formation shear only with all other parameters
fixed. Figure 5(a) shows the sensitivity of this one-parameter
inversion as a function of input dispersion shift upwards and
downwards by small values of slownesses at all frequencies.
Notice that when there is no shift in the input dispersion data,
the error in the inverted formation shear slowness goes to zero.
However, when dispersion shifts are introduced, the resulting
ersors increase quickly. This is because the Stoneley disper-
sion is more sensitive to the borehole fluid slowness rather
than the formation shear slowness as shown in Figure 4(a).
Figure 5(b) shows sensitivity of the two-parameter inversion
where we invert for both the formation shear and borehole fluid
slownesses. Again, the inverted slownesses are exact when
the input dispersion is accurately known. Even with $10 \mu s/ft$
slowness shift in the dispersion data, the relative error in the
estimated formation shear slowness is within $5\%$.

![Figure 6: Sensitivity of the formation flexural (a), and pipe
flexural (b) dispersions to small perturbations in model param-
eters; Relative error in the inverted formation shear as a func-
tion of uniform shift (or error) in the input formation flexural
c, and pipe flexural (d) dispersions; Relative errors in the in-
verted formation shear and mud compressional slownesses as
a function of uniform shift (or error) in the input formation
flexural (e), and pipe flexural (f) dispersions.](image)

![Figure 7: (a) Relative error in the inverted formation shear
slowness as a function of uniform shift (or error) in the input
quadrupole dispersion; (b) Relative errors in the formation
shear and mud compressional slownesses as a function of uni-
form shift (or error) in the input quadrupole dispersion.](image)
When inverting the dipole dispersion data in a fast formation, we have two flexural dispersions, one is dominated by the formation properties, another one is largely dependent on the drill collar properties. Figures 6(a) and 6(b) show their sensitivity functions, respectively. As the formation flexural is highly sensitive to the formation shear rather than other parameters in the model, the robustness of the one-parameter inversion (Figure 6(c)) is as good as the two-parameter inversion. (Figure 6(e)) for inverting the formation shear, but the latter algorithm gives us the advantage to infer the borehole fluid slowness correctly as well. On the other hand, when the tool flexural is highly sensitive to the fluid slowness, the two-parameter inversion is much more robust than the one-parameter inversion as shown in Figures 6(d) and 6(f).

Figure 8: Slow formation: (a) Sensitivity of the Stoneley dispersion to small perturbations in the model parameters; and (b) Relative error in the inverted formation shear slowness as a function of uniform shift (or error) in the input Stoneley dispersion data. (c) Sensitivity of the flexural dispersion to small perturbations in the model parameters; and (d) Relative errors in the inverted formation shear and mud compressional slownesses as a function of uniform shift (or error) in the input flexural dispersion data. (e) Sensitivity of the quadrupole dispersion to small perturbations in the model parameters; and (f) Relative errors in the inverted formation shear and mud compressional slownesses as a function of uniform shift (or error) in the input quadrupole dispersion data.

The quadrupole dispersion in a fast formation is sensitive to both the formation shear and borehole fluid slownesses (Figure 4(b)). Therefore, the inversion robustness is moderately improved when we invert for both parameters simultaneously (Figure 7(b)) compared to the one-parameter inversion algorithm shown in Figure 7(a).

In the case of a slow formation, the Stoneley, dipole and quadrupole dispersions are all highly sensitive to the formation shear slowness as shown in Figures 8(a), 8(c), and 8(e). Hence, both the one-parameter and two-parameter inversion algorithms yield excellent results. We illustrate results from the two-parameter inversion algorithm in Figures 8(b), 8(d), and 8(f).

Inversion of the Stoneley and quadrupole dispersions for the formation shear slowness

We have also tested our algorithm for estimating the formation shear slowness from the monopole Stoneley and quadrupole dispersions obtained from the processing of sonic data recorded by a prototype LWD-sonic tool. The LWD-sonic data was acquired from a field in central Texas. The recorded waveforms were first processed to obtain borehole dispersions using an automatic dispersion extraction algorithm (Valero et al., 2008; Bose et al., 2008). We have used our two-parameter inversion algorithm to produce logs of formation shear and borehole fluid slownesses at all depths. To check the accuracy of our inverted slownesses, we re-calculate modal dispersions using the inverted parameters, and compare them with measured dispersions obtained from the recorded waveforms. Overall, excellent agreement is obtained for most of the depths between the measured and modal dispersions calculated from the inverted parameters. Figure 9 illustrates two typical examples from the inversion of monopole and quadrupole dispersions for the formation shear slowness. The red stars are the input dispersion samples used in the inversion algorithm. The blue line is the re-calculated dispersion computed using the inverted slownesses (black dashed line). Measured dispersions obtained from the recorded waveforms are also shown to confirm its agreement with the model dispersions using the inverted parameters.

Figure 9: The Stoneley (a) and quadrupole (b) dispersions computed from the inverted parameters agree with those from measured dispersions obtained from the processing of recorded waveforms from a LWD-tool.

CONCLUSIONS

We have presented a new workflow to estimate the formation shear and borehole fluid compressional slownesses from a bandlimited Stoneley, flexural or quadrupole dispersion. Unlike a conventional DSTC algorithm for estimating the formation shear slowness, this new workflow is independent of overlapping arrivals in the precessing time window. However, this workflow can be used only after recorded waveforms have been successfully processed to isolate both the non-dispersive and dispersive arrivals in the wavetrain. In addition to the inversion efficiency of the proposed workflow, a proper selection of dispersion bandwidth mitigates any potential influence of near-wellbore alteration.
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REFERENCES


