Mechanical compaction in heterogeneous clastic formations from plastic-poroelastic deformation principles: Theory and modeling results

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Summary

A unified approach for mechanical compaction modeling in clastic basins is presented. The modeling approach is consistent with natural strain increment formulation often used in plasticity theory. Laboratory and field estimates of constitutive plastic deformation relations for sand-shale mixtures are used in a numerical model that generates estimates of porosity in 3D under various pore pressure, shale content, and loading scenarios. These estimates can be used in a variety of settings to predict various basin and reservoir properties associated with different loading conditions and/or sedimentation processes.

Introduction

Mechanical compaction or loss of porosity due to increase in effective stress is a fundamental geological process that governs many of the rock elastic and transport parameters of interest in exploring and developing subsurface reservoirs. Mechanical compaction is an irreversible process and can be viewed in terms of material yield (Goulty, 2004). The porosity loss is a plastic deformation process, and the stress path is a major factor in defining the material deformation due to irreversible failure.

Compaction modeling provides a working tool to address various exploration and development problems such as velocity modeling, AVA analysis, and more (Avseth et al., 2003). While many empirical depth trends have been suggested to model porosity change with respect to depth below mudline (e.g., Ramm and Bjorlykke, 1994), these empirical relations fail in places with high pore pressure and do not account for the physics of the deformation.

Compaction is a relation between effective stress and porosity loss. Examples of simple compaction laws have been suggested in the literature (Dutta, 2002; Sayers, 2006). Such relations were used to relate vertical effective stress to porosity and are often used for pore pressure prediction in regions where pore pressure buildup is governed by compaction disequilibrium process. However, it is also well known that compaction formulas are lithology dependent, and thus, often different compaction parameters are needed to describe shale and sand compaction.

In this paper, I present a generalized approach for compaction modeling. I present a fundamental differential equation for compaction modeling that is equivalent to natural strain increment formulation often used to describe stress-strain relations in plasticity theory. I solve this equation using differential constitutive plastic deformation relations relating differential porosity loss to differential effective stress increment for sand-shale mixture, and present example for modeling porosity in heterogeneous sand-shale sediments with variable pore pressure conditions. Applications of this model include modeling velocities and solving pore pressure estimation problems in various shale/sand mixtures.

Theory

1. Differential compaction and natural strain increment.

Following Goulty (2004), porosity loss is a plastic deformation process where the loss can be viewed as plastic strain in a representative elementary volume of sediment. When dealing with mechanical compaction, the elastic strain in the sediment can be neglected with respect to the plastic deformation associated with porosity loss. The differential volumetric plastic strain in sediment \( \varepsilon^p \) can be defined in terms of differential porosity loss as:

\[
de\varepsilon^p = d\phi / \phi .\]  

(1)

A constitutive relation in large deformation theory can be defined using a natural strain increment (Malvern, 1969) as:

\[
de\varepsilon^p = f(\sigma, \varepsilon, t) d\sigma / dt .\]  

(2)

An example for applying a natural strain increment to porosity loss can be demonstrated using simple compaction laws. Following Dutta (2002) and Sayers (2006), I consider the following three popular compaction laws often used to relate porosity to effective stress:

Equation 3a below is often known as Athy’s compaction, equation 3b relates stress to void ratio and can be attributed to Skempton, and equation 3c has been used by Sayers.

\[
\phi = \phi_0 \exp(-K\sigma_{eff}) \quad (3a)
\]

\[
\sigma_{eff} = \sigma_0 \exp(-\beta \phi(1-\phi)) \quad (3b)
\]

\[
\sigma_{eff} = \sigma_0 (1-\phi/\phi_0)^C \quad (3c)
\]

Here, for Athy’s relation, the compaction parameter is \( K \); for Skempton’s relation, the parameter is \( \beta \), and for Sayers,
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The exponent is $C$. $\phi_0$ is the critical porosity and $\sigma_0$ is the stress needed to get the sediment into zero porosity. In general, these model parameters are lithology dependent. Also, it is important to note that the relations above are not general and are stress-path dependent. In many cases, they were used under “primary loading” or “normal trend” conditions, often related to the loading experiment where either uniaxial loading or hydrostatic loading conditions are assumed.

In equation 4, I present the differential form of equation 3.

$$d\phi/\phi = -K d\sigma_{eff}$$

$$d\phi/\phi = -d\sigma_{eff} \left(\frac{\sigma_{eff} \beta (1-\phi)}{\sigma_0 (1-\phi_{0})^{C-1} \phi}\right)$$

$$d\phi/\phi = d\sigma_{eff} \left(\frac{C \phi_0 \sigma_0 (1-\phi_{0})^{C-1} \phi}{C \phi_0 \sigma_0 (1-\phi_{0})^{C-1} \phi}\right).$$

Equation 4 is equivalent to equation 2 when considering a steady-state solution for the volumetric plastic strain in compacting sediments. Note also that, while equations 3 and 4 have different functional dependencies for the different compaction models, they all behave similarly and can be calibrated to produce almost identical porosity-stress curves within the range of porosities between 0.4 and 0.1. This is demonstrated in Figure 1.

Figure 1: Comparison of three compaction models in equation 3. Note that they all produce very similar curves for porosities in the range of 0.4 to 0.1.

2. Compaction of sand and shales in the laboratory and well log data.

In Figure 2a, I present laboratory data from Yin (1992) where porosity is measured for a sand clay mixture at different states of confining stress. In Figure 2b, the data are plotted in the stress-porosity space after normalization with respect to critical porosity; the color bar is the clay content (vcl). Note that the stress–porosity curve is a strong function of vcl for a porosity range less than 40%, while it remains almost constant for vcl larger than 40%. This can be fitted using one of the constitutive models of equation 3, where fitting parameters are derived using a least-squares regression, and the parameters accommodate changes in clay volume while assuming a specific primary loading path associated with the laboratory experiment. I use critical velocity vs. the vcl relations of Yin (1992) and optimally fit parameters in equation 3a to describe the model as a surface in the stress-porosity-vcl space (Figure 4) associated with Yin’s data. Similar relations can be fitted to well log observations when effective stress estimates are available. In Figure 6, I show the results of fitting data from seven wells in the Gulf of Mexico (GoM) presented by Bachrach et al. (2007). Although I present here the deterministic model, the model prediction error can be easily accommodated when moving to a stochastic modeling mode, which will not be discussed here.

The differential form presented in equation 4 is lithology dependent but can also be path dependent. For example, considering equation 4a, the lithology dependent strain increment can be formulated using a relation of the type

$$de^p = d\phi/\phi = -K (vcl) d\sigma_{eff},$$

Figure 2: Sand-shale mixture compaction experiment (Yin, 1992). Top: Porosity vcl data where color bar is the confining pressure. Bottom: Same data presented where the y-axis is porosity normalized by critical porosity and the x-axis is confining stress. Color bar is vcl. Note that compaction slope varies considerably when vcl ranges from 0 to 40%, while for 40-100%, the vcl slope is almost constant.
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where the model parameter $K(vcl)$ is now $vcl$ dependent. Natural strain increments can be used for different stress paths, which will translate in this case into a path-dependent compaction parameter, i.e., $K=K(path)$. In the simple case of vertical loading and unloading, two compaction parameters can be used to model the stress-path differential effect.

3. **Fundamental equation for porosity loss under vertical loading at various pore pressure conditions using a natural strain increments concept.**

Porosity reduction in sedimentary basins can be viewed as following primary loading curves where the vertical loading of sediments increase the effective stress and the plastic deformation of the sediment associated with the volumetric porosity loss. To model porosity evolution in sedimentary basins, we must add spatial (i.e., depth and space) aspects to the deformation. This can be done using the following poroelastic equations:

The effective stress in relation to overburden stress and effective pressure using Terzhagi's relation:

$$\sigma_{eff} = \sigma_{ob} - \sigma_{P}. \quad (6)$$

Density is directly related to porosity. If we neglect to first order the variation in grain density with respect to depth, we can define the density porosity relation given by:

$$\rho'(z) = \phi(z)\rho_w + (1 - \phi(z))\rho_g. \quad (7)$$

The overburden stress under the primary loading path is given by:

$$\sigma_{ob}(z) = \rho_w g z_0 + \sum_{z_0}^{z} \rho(z')dz'. \quad (8)$$

Combining equations 6, 7, and 8, we can write the generalized relation between effective stress, porosity, and pore pressure as:

$$\sigma_{eff}(\phi) = \rho_g g(z - z_0) + (\rho_w - \rho_g) g \int_{z_0}^{z} \rho(z')dz' - P_f(z). \quad (9)$$

A differential form of equation 8 is given by:

$$\frac{\partial \sigma_{eff}}{\partial \phi} \frac{\partial \phi}{\partial z} = \rho_g g + (\rho_w - \rho_g) g \phi \frac{\partial P_f}{\partial z}. \quad (10)$$

I note that the differential form of equation 10 is the one that is best suited for constitutive law in terms of natural strain increment. Using equation 4, we can write the differential equation in terms of the differential constitutive compaction law. Moreover, as the equation is posed as an ordinary differential equation (ODE), one can integrate equation 9 using non-linear ODE solvers. For a depositional sequence with a variable shale volume, one can use laboratory measurements such as these described by a model calibrated on laboratory or field measurements such as those presented in Figure 3.

![Figure 3: Top: The stress-vcl-porosity surface derived by modeling Yin’s data with variable compaction parameters. Bottom: Example of modeling results of well log data published by Bachrach et al., (2007).](image)

It is also important to note that the pore pressure must be explicitly given to properly model compaction. When pore pressure is not known, one can predict compaction under various pore pressure regions such as hydrostatic or typical basin profile. It is shown here that pore pressure must be assumed or known when modeling compaction.

**Synthetic example**

To demonstrate how the model works, consider a 2D layer with a variable shale-sand sequence imbedded between two shale layers as presented in Figure 4a. The
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Top of the shale layer is assumed to be the mudline. A typical GoM pore pressure is assumed, but an elevated pore pressure zone (pressure cell) in the middle of the section is made to demonstrate the effect of lateral changes in pore pressure. The sand volume and pore pressure profile are used to generate porosity maps by solving the porosity evolution non-linear differential equation 10. In Figure 5, I present the estimated spatially varying porosity derived by solving equation 10 for the differential constitutive law 3a using variable parameters that were used in calibrating laboratory and GoM data (Figure 3). It is easy to see the effects of the high pore pressure and the shale/sand variation on the porosity profile in space. High pore pressure prevents fast compaction and sand zones compact less than shale zones.

In Figure 6, I present a velocity profile derived using porosity-velocity relations for sand-shale mixtures using the porosity data. The velocity is derived using a simple effective medium model for sands and shales. It demonstrates the potential of this method to generate initial models for velocity that can be used for velocity model building, prior to AVA analysis and more.

Discussion and conclusions

I present a formulation for mechanical compaction modeling based on a general differential equation that captures differential stress-strain plastic deformation relations for sand/clay mixtures. The model formulated using effective stress enables us to account for various pore pressure and lithology variation in space. Results are consistent with poroelasticity in terms of effective stress, pore pressure relations, and capture irreversible deformation processes. The theory can be used to model various loading scenarios and to predict porosity and seismic velocities.

Figure 5: Porosity modeling results using equation 9 and input data presented in Figure 4.

Figure 6: Seismic velocity estimation using porosity-velocity relations for sands and shaley sand mixtures.

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REFERENCES


