Wave equation receiver deghosting: a provocative example
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Summary
The problem of counteracting the effects of the receiver ghost in marine data has generally been addressed either by making complementary measurements such as hydrophone pairs or hydrophone and particle velocity sensors or by estimation of the effect through the so-called ghost model.

Here, we discuss a technique based on the wave equation that accomplishes up/down wavefield separation based on a single measurement that does not rely on the spectral ghost model and does not rely on statistical or other means of estimating missing frequencies. While it is a distinct advantage to record multiple measurements, such systems are limited in availability and, moreover, significant legacy data exist that could benefit from deghosting. We briefly review the theory for wave equation receiver deghosting – first presented by Beasley et al., (2013) – and then present examples that confirm the accuracy of the approach. We illustrate a significant difference between this approach and many other approaches that use single measurements. Using the notch frequency in a mono-frequency plane wave example, we show that our approach accurately reconstructs the upgoing wavefield without benefit of information from other frequencies; a task that we expect would pose challenges to other single-measurement approaches.

Introduction
The problem of accounting for the amplitude and phase distortions introduced by the so-called ghost effect where up and downdowngoing wavefields interfere was first studied in the context of sources by Van Melle and Weatherburn (1953). They showed that by using more than one source with a delayed firing pattern, it was possible to mitigate the ghost effect. Later, this concept of multiple measurements was adopted for marine receivers in the context of hydrophone/geophone pairs and over/under hydrophones (Berni, 1984; Sønneland et al. 1986) and for ocean bottom cables (OBC) (Barr and Sanders 1989). While OBC systems gained rapid commercial acceptance, commercial systems employing multimeasurements are more recent (Carlson et al. 2007; Robertsson et al. 2008).

Lacking multimeasurements for marine data, researchers have devised a number of ways to mitigate the receiver ghost problem through either specialized acquisition (Ray 1982; Dragoset 1991; Soubaras 2010) or through estimation procedures based on the either the spectral ghost model or estimation of a second measurement (Robertsson and Kragh 2002; Ferber et al. 2012).

Our approach requires no special acquisition and uses only a single measurement. Assuming we know receiver depths, water velocity and the water surface properties, we use the wave equation to simulate propagation of the up- and downdowngoing wavefields between the receivers and the water surface to accomplish wavefield separation. A key element to our approach is that we use the fact that, assuming noise and direct arrivals are removed, the upcoming wavefield is causal with respect to the downdowngoing wavefield and thus supplies an initial condition to begin the wave propagation.

Theory
A more detailed discussion of the theory has been presented (Beasley et al., 2013) but here we provide a kinematic discussion for motivation. We begin by examining the downdowngoing wavefield associated with a seismic source at position $i$ on the streamer at time $t$ denoted $D_i(t)$. As illustrated in Figure 1, elements of the downdowngoing wavefield are created by earlier instances of the upcoming wavefield $U$ (neglecting direct arrivals and noise). The total downdowngoing wavefield is the superposition of all such contributions and can be written as

$$D_i(t) = \sum_j A_{ij} U(t-\Delta t_{ij})$$  \hspace{1cm} (1)

where $\Delta t_{ij}$ is the traveltine for the upcoming wave recorded at location $j$ to reach location $i$ (as a downdowngoing wave) and $A_{ij}$ is an amplitude term. The total wavefield $W$ at time $t$ is the sum of the up and downdowngoing parts so $W(t) = U(t) + D(t)$ and substituting for $D$ from equation (1) we find that

$$W(t) = U(t) + \sum_j A_{ij} U(t-\Delta t_{ij})$$  \hspace{1cm} (2)

Solving for $U$ in equation (2) gives the relation

$$U_i(t) = W(t) - \sum_j A_{ij} U(t-\Delta t_{ij}).$$  \hspace{1cm} (3)

Equation (3) suggests that the upcoming wavefield can be computed iteratively by first evaluating a Kirchhoff integral over earlier (and previously computed) values of $U$ which is then subtracted from the recorded wavefield $W$ which yields the next value of $U$. In practice, it may be preferable to recast the problem as iterative extrapolations of the up- and downdowngoing wavefields. This approach, similar to migration, formulates the problem as a system of wave equation operators (such as RTM, phase shift or the like) coupled by the surface reflection boundary condition. The reader is referred to Beasley et al. (2013) for a more comprehensive exposition of this approach.
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Figure 1  Downgoing wavefield expressed as the composition of earlier elements of the upcoming wavefield. In general, the downgoing wavefield is the sum of all such components.

Synthetic Data Examples

We presented the following synthetic example modeled from the Gulf of Mexico SEG advanced model (SEAM) to illustrate the effectiveness of wave equation deghosting in the presence of complex geology (Beasley et al. 2013). Figure 2 shows the total wavefield on the left, the deghosted wavefield in the center and the error, which we define as the difference between the deghosted data and data modeled without the free surface boundary condition.

The 50 m cable depth and 1500 m/s water velocity used in the model imply a notch frequency of 15 Hz. Figure 3 shows comparisons of averaged temporal frequency spectra computed in the box annotated on the input data in Figure 2. The spectra indicate that the notch frequency and those nearby have been reconstructed nearly perfectly. This, along with the difference section in Figure 2, indicates that the wavefield separation and deghosting is accurate.

However, this being noise-free synthetic data, it is likely that other approaches to deghosting single-measurement data would also fare well in this test. Moreover, given the different approach we have taken to the deghosting problem, one might wonder if we have come upon simply another derivation of the familiar spectral ghost model. In other words, is there really a difference in wave equation deghosting and other ghost-model based approaches?

To answer this question, we turn to a very simple synthetic data example consisting of a mono-frequency plane wave. Figure 4 shows the upcoming, downgoing and total wavefields for our model consisting of a series of continuous, mono-frequency, flat plane waves with receiver depth, water velocity and notch of 50m, 1500 m/s and 15 Hz respectively, as in the previous example. We have chosen 15 Hz as the mono-frequency in the plane wave.
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Figure 4 Mono-frequency plane-wave data modeled at the notch frequency of 15 Hz. The up- and downgoing wavefields are summed to produce the total and cancel except for the causal part, which is the first period of the upcoming wavefield.

Figure 4 shows the upgoing and downgoing wavefields which were modeled independently and then added together to generate the total wavefield. Recording was started before the upcoming wavefield reached the receivers so, apart from slight edge effects, the total wavefield consists of only the first period of the upgoing wavefield – the causal part. As expected, the rest of the recorded wavefield is zero. Figure 5 shows the result of deghosting the input data with wave equation deghosting and the error. After subtracting the dehosted data from the modeled upcoming wavefield, we see it has produced a nearly perfect result.

Discussion and Conclusions

The mono-frequency example shown above would likely pose significant challenges for other approaches to single measurement deghosting. In particular, with only the causal part of the wavefield present and no other frequencies from which to estimate the missing data, it seems nearly impossible, however, as we are not able to test all other approaches, we acknowledge that perhaps there may be methods and techniques unknown to us that can deghost this mono-frequency data. Of course, in practice, other frequencies would be present and would aid in the reconstruction of the notch.

While this is not a realistic example in terms of real recorded data, it nevertheless illustrates our main contention which is wave equation deghosting is fundamentally different from other methods based on the spectral ghost model known to us.

We have discussed before (Beasley et al. 2013) that while promising, this approach has significant challenges in
application to real data. As it involves wave propagation, it requires unaliased data and hence requires fine receiver sampling. Cables that use arrays of hydrophones will mix the data and thus single sensor recording is preferred; for 3D deghosting, receivers must also be sampled finely cross line. Water velocity, receiver depth and receiver positions must be known. Moreover, the influence of noise will disrupt the process in a predictable way. Edge effects and instabilities (although we have not observed any) are possible. We should point out, however that all of these issues are likely issues in other approaches to deghosting so wave equation deghosting is subject to many of the same issues as conventional deghosting approaches.

Despite this daunting array of issues we believe that there is merit in pursuing wave equation deghosting. First of all, it brings in the physics of the problem which is always beneficial. Since it handles all dips, it is possible that there are issues of resolution that may be addressed by using the wave equation. Issues such as water velocity variation, varying sea surface and reflection characteristics could be addressed in a natural manner. We may derive methods for estimating these quantities directly from the data, much as we do for migration velocity today. Finally, we don’t require any specific acquisition geometry such as slanted cables nor do we employ statistical or other procedures to estimate the unrecorded data. Instead we use the wave equation and causality of the upcoming wave field to reconstruct the full upgoing wavefield. Clearly, wave equation deghosting offers the promise of extending the benefits of deep-towed streamers to conventional recording systems but does it obviate the need for multimeasurement systems? We think not. We believe that while it is possible to compute the upgoing wavefield there are significant benefits to having complementary measurements. We believe that it is complementary to multimeasurement data particularly in the case of 3D deghosting.

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EDITED REFERENCES
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