Can we distinguish TTI and VTI media?
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Summary
Modern depth imaging requires the ability to build transversely isotropic models of the subsurface. Vertical transverse isotropy (VTI) assumes that the symmetry axis is vertical, whereas the more general case of tilted transverse isotropy (TTI) may have a symmetry axis away from vertical. While VTI is a simpler representation, TTI may be more geologically plausible for sedimentary formations such as shales. We examine a simple TTI model and demonstrate that even in the presence of additional well information such as a checkshot, obtaining a symmetry axis orientation from data alone may be ambiguous. Therefore additional geological information from wells, basin evolution and geomechanics may be required.

Introduction
The majority of present day depth imaging is performed using the assumption of vertical transverse isotropy (VTI) for subsurface formations. However there is growing evidence that TTI models may be a more appropriate description (Audebert et al, 2006). A TTI model requires two additional angle parameters to specify the orientation of the symmetry axis. If the subsurface is TTI but we ignore it, then we expect to see unexplained azimuthal anisotropy as well as significant lateral mispositioning of imaged reflectors. However in practice distinguishing between VTI and TTI models is a matter of art and a priori geological assumptions. In this study we attempt to take a data-driven approach and to analyze these decisions and their implications on a synthetic dataset. The dataset consists of narrow-azimuth surface seismic data and a vertical checkshot, modeled for a layered TTI media. We show that even with the vertical well data the TTI model cannot be fully resolved. We further demonstrate that the VTI model can explain the data equally well. Yet, the VTI model is incorrect which manifests itself in lateral mispositioning and azimuthal variation in velocity.

Synthetic example
Let us consider a simple deepwater model composed of layered TTI sediment with a constant symmetry-axis tilt of 45 degrees. The model has smooth vertical variation of velocity with two low-velocity zones (Figure 1a). Reflected events from multiple interfaces of density contrast are located every 200 m. Maximum offset is 12 km. A prestack gather computed with anisotropic ray tracing is shown in Figure 1b. Thomsen parameters can be found in Figure 2. We apply anisotropic reflection tomography described by Woodward et al (2008). Tomography will solve for a local anisotropic model using joint inversion of seismic and well data. Well data is represented by the checkshot survey acquired along the vertical well in the range of depth from 1.5 km to 11 km every 50 m. Before inversion we apply a mute of 50 degrees to the data which limits useable offsets to less than 8 km for a depth of less than 6 km. At depths below 8 km angles are capped at 40 degrees or less even for maximum offset.

We consider two scenarios in the inversion:
• invert for a TTI model and assume that the tilt of the symmetry axis is known
• assume vertical symmetry axis and invert for a VTI model.

TTI inversion of seismic and vertical checkshot data
In the first scenario we assume that tilt of the symmetry axis is known. Since vertically propagating velocity in TTI media is a function of all the parameters, then we jointly invert seismic and checkshot data for three parameters ($V_p$, $\varepsilon$ and $\delta$) around the well. It will be proven later that TTI inversion is not unique in this case. If our initial model is isotropic (TTI model with zero anisotropy) with velocity taken from stacking velocity analysis, then tomography recovers one of the possible TTI models (Figure 2) that fits the checkshot data within 10 ms accuracy and flattens the image gathers, but has geologically implausible $\varepsilon$ and $\delta$. 

Figure 1: Symmetry-axis velocity profile (a) and pre-stack gather (b) for a deepwater model represented by horizontally layered TTI medium with many density reflectors; symmetry axis is tilted by 45 degrees away from vertical for all layers; water depth is at 1500 m.
Figure 2: Results of a three-parameter inversion ($V_{P0}$, $\varepsilon$, and $\delta$) of seismic and vertical checkshot data for TTI model assuming that orientation of the symmetry axis is known correctly. Velocity and anisotropy profiles after each iteration are shown together with initial and true models: (a) update in velocity shown as a difference between current velocity at each iteration and initial velocity profile; (b) $\delta$; (c) $\varepsilon$; (d) $\eta$. Note that whereas parameter combination $\eta \approx \varepsilon - \delta$ is relatively well constrained, tomography recovers one of the equivalent models with incorrect $V_{P0}$, $\delta$, and $\varepsilon$. Note that parameter $\eta$ in our case plays the same role as $\delta$ in VTI case relating vertical velocity and $V_{NMO}$ according to equation (4). Even though vertical velocity ($V_V$) is constrained by the checkshot, we are unable to resolve $\varepsilon$ and $\delta$ individually, because in our case both short- and long-offset moveouts are controlled by $\eta$ (see equations 3 and 4).

Figure 3: Results of a two parameter inversion ($\varepsilon$ and $\delta$) of seismic data after assuming a VTI model and fixing vertical velocity to correct values based on the checkshot. Anisotropy profiles after each iteration are shown together with initial and true models: (a) $\delta$; (c) $\varepsilon$; (d) $\eta$. Note that recovered $\delta$ is quite close to the true $\eta$, whereas $\varepsilon$ is close to zero.
VTI inversion

In this scenario we assume vertical symmetry axis and invert vertical symmetry-axis velocity directly from checkshot traveltimes. After fixing vertical velocity we perform a two-parameter inversion ($\delta$ and $\varepsilon$) of the seismic data and recover the following profiles (Figure 3). This solution has positive $\delta$ but almost zero $\varepsilon$ and therefore negative $\eta$.

What are the implications?

Both models produce flat image gathers (Figure 4) as well as provide good fit to the vertical checkshot. Therefore data does not disqualify any of the two models. Does it mean we should opt for a simpler model, i.e. VTI, that fits the data? No. Good kinematic fit (good data focusing) is not sufficient for correct positioning: wrongly using a VTI model that fits seismic moveout and VSP traveltimes will result in mispositioning and apparent azimuthal anisotropy. For example, if we were to image even our flat TTI data with the VTI instead of the TTI model, any amplitude anomalies along the flat reflectors would be laterally mispositioned. VTI migration of our data will always propagate energy symmetrically around a vertical reflection axis: for any offset, an image point will always fall midway between the source and receiver. TTI migration of the same data will propagate energy asymmetrically, due to the different velocities and consequently unequal angles of incidence and reflection around the vertical axis (Figure 5a). Because of this asymmetry, image points are displaced to the left of their common midpoint locations by up to 700 m (Figure 5b).

Both inversion scenarios result in models that have unusual anisotropy profiles that are unlikely to be geologically plausible. Interestingly the VTI model looks less strange because it has positive and reasonable $\delta$; only near-zero $\varepsilon$ and negative $\eta$ may raise our concern. If maximum offset is reduced, then $\varepsilon$ may no longer be constrained and thus this suspicion may be ignored. In contrast, the TTI model looks more suspicious with both $\varepsilon$ and $\delta$ being negative and with $\delta$ being less than $\varepsilon$. Let us find an explanation

![Figure 4: Common-image gathers around the well for different depth models: (a) VTI model, (b) alternative TTI model obtained by inversion; (c) true TTI model used for generating the synthetics. Note that gathers are equally flat on all three panels. Checkshot traveltimes are also fit within 10 ms for all models (not shown). There is only a small mistie between the first two models and the true model increasing with depth caused by imperfect fit of the checkshot data.](image)

![Figure 5: Location of subsurface reflection points and angles in the true TTI model as predicted by ray modeling: (a) deviation of opening angle bisector from vertical direction; angle bisector divides opening angle between incident and reflected rays in half. Note that for VTI media angle bisector is always zero, whereas for TTI media we observe consistent shift of the angle bisector for all offsets including zero offset. Both attributes are color-coded by source-receiver offset. Observe the strong correlation between (b) and the anisotropy profiles (Figure 2). (b) Shift of the actual reflection point compared to the midpoint location between source and receiver.](image)
of such results using theoretical analysis of seismic signatures.

**Weak-anisotropy analysis of TTI inversion**

In order to understand the TTI inversion it is instructive to obtain weak-anisotropy expressions for all P-wave TTI signatures at hand. For a single horizontal TTI layer with a 45 degree tilt of the symmetry axis these signatures are expressed as follows (Tsankin, 2001; Pech et al, 2003):

\[
V_{\text{TTI}}^{\text{av}} = V_{\text{PV}}^{\text{TTI}} \left( 1 + 1.25 \mathbf{e}^{\text{TTI}} - 0.75 \delta^{\text{TTI}} \right),
\]

(1)

\[
V_{\text{TTI}}^{V} = V_{\text{PV}}^{\text{TTI}} \left[ 1 + 0.25(\mathbf{e}^{\text{TTI}} + \delta^{\text{TTI}}) \right],
\]

(2)

\[
\eta_{\text{TTI}}^2 = \frac{2\eta^{\text{TTI}}}{V^2_{\text{PV}}(V_{\text{PV}}^{\text{TTI}})^2},
\]

(3)

where \(V_{\text{PV}}^{\text{TTI}}, \mathbf{e}^{\text{TTI}}, \delta^{\text{TTI}}\) are the three independent Thomsen parameters that describe the TTI velocity field; \(\eta^{\text{TTI}} \approx \mathbf{e}^{\text{TTI}} - \delta^{\text{TTI}}\); \(V_{\text{PV}}^{\text{TTI}}\) denotes velocity in the true vertical direction; \(V_{\text{PV}}^{\text{TTI}}\) describes the moveout velocity for a horizontal reflector; \(\eta_{\text{av}}\) is a quartic moveout coefficient describing travelt ime behavior at long offsets. Various numerical coefficients arise after substituting values of zero for reflector dip and tilt of the symmetry axis (45 degrees). It is instructive to combine equations (1) and (2) and thus rewrite (1) in the following weak-anisotropy form

\[
V_{\text{TTI}}^{\text{av}} = V_{\text{PV}}^{\text{TTI}} \left( 1 + \frac{\eta^{\text{TTI}}}{\eta_{\text{av}}^{\text{TTI}}} \right),
\]

(4)

It is easy to see that three measurements \((V_{\text{PV}}^{\text{TTI}}, V_{\text{TTI}}^{V} \text{ and } \eta_{\text{TTI}}^2)\) do not constrain all three VTI parameters \((V_{\text{PV}}^{\text{TTI}}, \mathbf{e}^{\text{TTI}} \text{ and } \delta^{\text{TTI}})\), because equations (4) and (3) constrain only \(\eta^{\text{TTI}}\), whereas equation (2) constrains the remaining combination of all three desired quantities. In the absence of any additional information, tomography retrieves an equivalent model with correct \(V_{\text{TTI}}^{\text{av}}\) and \(\eta^{\text{TTI}}\) but incorrect individual parameters \(V_{\text{PV}}^{\text{TTI}}, \mathbf{e}^{\text{TTI}} \text{ and } \delta^{\text{TTI}}\).

**Weak-anisotropy analysis of VTI inversion**

Now let us fit the TTI model at hand by best possible VTI model. It is easy to see that \(V_{\text{PV}}^{\text{TTI}} = V_{\text{PV}}^{\text{TTI}}\) and comparing equation (4) with the VTI equation

\[
V_{\text{TTI}}^{\text{av}} = V_{\text{PV}}^{\text{TTI}} \left( 1 + \frac{\eta^{\text{TTI}}}{\eta_{\text{av}}^{\text{TTI}}} \right),
\]

(5)

we conclude that \(\delta^{\text{TTI}} = \eta^{\text{TTI}}\). Finally, from symmetry considerations we deduce that horizontal and vertical velocities are equal to each other \(V_{\text{PV}}^{\text{TTI}} = V_{\text{PV}}^{\text{TTI}}\) and therefore \(\mathbf{e}^{\text{TTI}} = 0\). Therefore we may conclude that the VTI medium with these parameters \(V_{\text{PV}}^{\text{TTI}} = V_{\text{PV}}^{\text{TTI}}, \delta^{\text{TTI}} = \eta^{\text{TTI}}\) and \(\mathbf{e}^{\text{TTI}} = 0\) has these three signatures identical to a TTI model: velocity along vertical Z-axis, moveout velocity, and horizontal velocity. This explains excellent fit of the same data by TTI and VTI tomographic solutions.

**Discussion**

While 1D analysis can guide us on what to do in a simple case, in realistic 3D settings with structure it is not applicable. Although negative Thomsen parameters were a red flag in our example, in other cases positive but incorrect values of Thomsen parameters can be obtained when a VTI model is used to fit a TTI subsurface. One possible approach for choosing between a VTI and a TTI model may be to perform uncertainty analysis to understand the non-uniqueness of the problem and to characterize the parameter null space (Osypov et al., 2008). Such a study may guide selection of a model from a set of seismically equivalent choices to the one that maximizes geological plausibility. It may also indicate what other information or measurement would further constrain the solution. This additional information does not have to be excessive. For example, if correct delta profile is brought in and fixed for the TTI case, then tomography correctly recovers two remaining parameters.

**Conclusions**

We examine one of the key decisions in model building: whether to use a VTI or TTI parameterization. We demonstrate that for a general TTI media when symmetry-axis tilt is not related to bedding, such a decision will be difficult to make based on data alone. In particular we analyzed a realistic dataset for horizontally layered TTI with a constant tilt of 45 degrees. We found that estimation of the anisotropic velocity field for such a model is non-unique even if we assume knowledge of the tilt and velocity profile from a vertical well. A whole series of equivalent TTI models can fit the seismic and well data. If we start from a zero anisotropy model, then joint tomographic inversion for three TTI parameters finds one of the equivalent models with negative Thomsen parameters. On the other hand, if one assumes VTI symmetry then the same dataset of seismic and checkshot data can be fit with a VTI model. Theoretical analysis of the seismic signatures explains the observed ambiguity and suggests that Thomsen’s \(\delta\) for such a VTI model is equal to the parameter \(\eta\) in the TTI model. Therefore data-driven approaches recover two incorrect models – one VTI and one TTI – that both fit the seismic and well data, but with geologically implausible values of anisotropy. To arrive at the correct TTI model one has to bring additional information of some sort. For example, this could be velocity from a deviated well or Thomsen’s \(\delta\) profile from an offset well. Another resolution is to analyze the null-space of the solution and to make a selection of anisotropic parameters based on a priori information from the area or rock physics measurements.
EDITED REFERENCES
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REFERENCES