Stress induced seismic velocity anisotropy study
Bin Qiu, Mita Sengupta, Jorg Herwanger, WesternGeco/Schlumberger

Summary
A method of calculating stress induced seismic velocity anisotropy is presented and studied. In a first step an effective stiffness tensor of a stressed rock is calculated using third order elasticity (TOE) theory. In a second step, anisotropic seismic P- and S-wave velocities are calculated from the effective stiffness tensor using Tsvankin’s notation, which is an extension of Thomsen’s weak anisotropy notation. This method is used to study anisotropic velocity changes due to changes in hydrostatic, uni-axial and tri-axial stress using stress-sensitivity (third-order) parameters given in the literature. The computations show, in agreement with experimental observations, that the strongest P-wave velocity increases are observed in the direction of the largest stress increase. For S-waves, the largest velocity increase is observed for S-waves polarized parallel to the direction of maximum stress-increase. Therefore, S-waves have the potential to be used as a tool to monitor horizontal stress changes.

Introduction
Knowledge of the subsurface stress-state is an important factor of oilfield management. Knowledge of stress conditions in the overburden and the reservoir can help to avoid drilling hazard and helps to manage reservoir production. Pressure drawdown during hydrocarbon production causes subsurface stress to change and seismic methods have the potential to monitor this change in stress state. In the last 5 years at least 15 fields were reported to show a stress-related time-lapse seismic response in the open literature (see Herwanger and Horne, 2009 for a table compiling the observations). Given that not all such observations are reported and that time-lapse seismic is not applied in all producing fields, the actual number of fields where time-lapse seismic stress monitoring can be used is far larger. The rock-physics relationship linking the observed velocity changes and the causative stress-changes is still an active area of research. The majority of reported field case studies assume that stress-induced velocity changes are isotropic. On the other hand, experimental evidence from laboratory measurements (Scott et al., 1993; Scott et al., 1998; Dillen et al., 1999) clearly shows that anisotropic velocity changes occur during non-hydrostatic stress changes.

In this study, we examine the effect of elastic deformation of rock for hydrostatic, uni-axial and tri-axial compression on anisotropic seismic velocity. First, we present a method of computing stress induced changes in the elastic stiffness tensor based on third-order elasticity described by Prioul et al. (2004). Second, we then present anisotropy parameters and P-wave, S-wave velocities for orthorhombic media (Tsvankin, 1997). These methods are then used to study anisotropic velocity changes for each of three different scenarios and the resulting velocity changes are plotted. The vertical velocity is shown to be sensitive to stress path and depends on both vertical and horizontal strain. For the same amount of vertical strain, vertical velocity changes are by a factor of two larger for hydrostatic compression than for tri-axial stress changes following a stress-path typical for overburden stretching. The results predict that the percentage change in velocity anisotropy is far larger than the strain percentage change that is linked to tensor stress change. Observations of anisotropic seismic velocity changes from reflection seismic data have the potential, given suitable data processing, to act as a remote stress-sensing tool.

Effective stiffness tensor of a stressed VTI medium

Figure 1: (a) Seismic velocity in an initial stress state is assumed to be anisotropic with VTI (vertical transversely isotropic) symmetry. Velocity is described by the vertical P- and S-velocity $V_p$ and $V_s$, and the Thomsen parameters $\epsilon$, $\delta$ and $\gamma$. We then apply stresses along the coordinate axes, indicated by the yellow arrows. (b) Stress-induced seismic velocities can show orthorhombic symmetry, when all three principal stresses are unequal. Velocity is described by the vertical P-velocity $V_p$ and the two vertical S-velocities and the Tsvankin parameters $\delta^{(1)}$, $\delta^{(2)}$, $\delta^{(3)}$, $\gamma^{(1)}$ and $\gamma^{(2)}$.

In this section, we show how to compute the elastic stiffness tensor, describing seismic P- and S-wave velocity, of a medium under a change in the static stress field using third-order elasticity (TOE) theory. The equations follow the description given in Prioul et al. (2004). In the initial stress state elastic wave propagation in the rock mass is described by vertical transverse isotropic (VTI) elastic symmetry (Figure 1a). Wave speed in the horizontal plane is constant, i.e. independent of the azimuth of the
propagation direction of the wave. Waves propagating in different directions in a vertical plane have different propagation velocities. Applying a tri-axial stress field (denoted by the three arrows in Figure 1a), causes anisotropic velocity changes. If the two horizontal stresses are equal (but different from the vertical stress change) the rock will retain its VTI symmetry. In the general case (all three principal stresses are different), the rock mass will exhibit orthorhombic symmetry (Figure 1b). Note that the direction of the applied vertical stress is aligned with the symmetry axis of the VTI medium. In the TOE theory applied here, the principal axes of the applied stress field have to be aligned with the symmetry axis of the elastic tensor describing the medium. Generalizations of the theory are available, but need more than the three stress-sensitivity parameters in the formulation used in this paper.

Procedure for computing stiffness tensor under changing tri-axial stress conditions

Rock is a non-linear medium, i.e. stress is related to strain in a non-linear fashion. Under external (static) stress, rock deforms, its stiffness changes, and the effective elastic stiffness tensor becomes a non-linear function of stress (Thurston, 1974). To make the computation more tractable, a Taylor series expansion of the effective elastic stiffness tensor \( C_{\text{eff}} \) at an initial stress state can be used to compute the elastic stiffness tensor \( C_0 \) at any different stress state. This is described by TOE rock-physics model (Prioul, 2004) in equation group (1):

\[
\begin{align*}
C_{11} &= C_{0111} + C_{111} E_{11} + C_{112} (E_{22} + E_{33}) \\
C_{22} &= C_{0111} + C_{111} E_{22} + C_{112} (E_{11} + E_{33}) \\
C_{33} &= C_{0111} + C_{111} E_{33} + C_{112} (E_{11} + E_{22}) \\
C_{12} &= C_{0111} + C_{111} (E_{11} + E_{33}) + C_{112} E_{22} \\
C_{13} &= C_{0111} + C_{111} (E_{11} + E_{22}) + C_{112} E_{33} \\
C_{23} &= C_{0111} + C_{111} (E_{22} + E_{33}) + C_{112} E_{11} \\
C_{36} &= C_{0111} + C_{111} E_{33} + C_{112} E_{11} + C_{113} E_{22} \\
C_{55} &= C_{0111} + C_{111} E_{22} + C_{113} E_{11} + C_{112} E_{33} \\
C_{44} &= C_{0111} + C_{111} E_{11} + C_{112} E_{22} + C_{113} E_{33}
\end{align*}
\]

with
\( C_{0i} \): Effective stiffness tensor of stressed rock;
\( C_{0i} \): Stiffness tensor of rock in initial stress state;
\( E_{ii} \): Principal strain related to principal stress; and
\( C_{iik} \): The TOE sensitivity parameters.

The equations state that the elastic stiffness tensor \( C_0 \) at any stress state can be computed from the elastic stiffness tensor \( C_{0i} \) in an initial (or reference) stress state, the strain changes \( E_{ii} \) caused by static stress changes and three independent stress sensitivity parameters \( C_{111} \), \( C_{112} \) and \( C_{113} \), with \( C_{112} = (C_{112} - C_{113})/2 \) and \( C_{113} = (C_{111} - C_{112})/4 \). In this study, the elastic stiffness tensor in the initial stress state is taken to exhibit VTI anisotropy. The equation (1) is a linearized form of the fully non-linear theory described in Thurston, 1974 and Sinha and Kostek, 1996. Note, in equation (1), only rock strain is used. It can be calculated from stress using inverse Hooke’s law (Herwanger and Horne, 2009). In this paper, although stress is mentioned frequently, strain is actually used in calculations.

Anisotropic parameters and p-wave velocity for orthorhombic media

A set of notations to describe anisotropic velocity in orthorhombic media was introduced by Tsvankin, 1997. It extended Thomsen’s weak anisotropy notation to describe the orthorhombic velocities. The notation provides expression to compute P-wave velocity as a function of polar angle \( \theta \) and azimuthal angle \( \Phi \) (denoted in Figure 2) from the stiffness tensor of an orthorhombic medium \( C_{\text{orthor}} \). Also S-wave velocities as function of polar angle \( \theta \) in two symmetry planes are provided. Here the elements of the \( C_{\text{orthor}} \) are computed from the TOE rock-physics model:

\[
C_{\text{orthor}} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\]

Figure 2: Nomenclature of velocities in an orthorhombic medium following Tsvankin (1997).

Figure 2 shows the nomenclature for velocities of an orthorhombic media according to Tsvankin (1997). Equation (2) computes P-wave velocity. Equations (3), (4) and (6) compute \( S_{11} \) and \( S_{12} \) velocity for the \([x1,x3]\) and \([x2,x3]\) symmetry planes, respectively:

\[
V_p(\theta, \phi) = V_{p0}\{1 + \delta \sin^2 \theta \cos^2 \phi + \epsilon \sin^2 \theta\}
\]
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\[ V_{st} (\theta) [\{x_1, x_2, x_3\} \text{plane}] = V_{50} \sqrt{1 + 2y^{(1)} \sin^2 \theta} \]

(3)

\[ V_{sv} (\theta) [\{x_1, x_2, x_3\} \text{plane}] = V_{sv0} \pm \sqrt{V_{sv0}^2 - \left(2y^{(2)} \cos \theta \sin^2 \theta \right)^2} \]

(4)

\[ V_{st} (\theta) [\{x_2, x_3\} \text{plane}] = V_{st0} \sqrt{1 + 2y^{(1)} \sin^2 \theta} \]

(5)

\[ V_{sv} (\theta) [\{x_2, x_3\} \text{plane}] = V_{sv0} \sqrt{1 + 2y^{(2)} \cos \theta \sin^2 \theta} \]

(6)

This description uses the following naming conventions:
- \( V_p \) — Vertical P-wave velocity with angle \( \theta \) to vertical axis 3 and angle \( \phi \) to horizontal axis 2;
- \( V_{50} \) — Vertical P-wave velocity in the x1-x3 plane with vertical axis 3 in symmetry plane 1-3;
- \( V_{sv0} \) — Vertical pseudo shear wave velocity polarized in the x1-x3 plane with vertical axis 3 in x1-x3 plane;
- \( V_{sv} \) — Vertical pseudo shear wave velocity polarized in the x2-x3 plane with vertical axis 3 in symmetry plane 2-3;
- \( V_{50} \) — Vertical pseudo shear wave velocity polarized in the x2-x3 plane with vertical axis 3 in x2-x3 plane;
- \( V_{sv} \) — Vertical pseudo shear wave velocity polarized in the x2-x3 plane with vertical axis 3 in symmetry plane 2-3.

The velocity and anisotropy parameters are related to the elements of the stiffness tensor by:

- \( \epsilon^{(1)} \) — VTI parameter \( \epsilon \) in x1-x3 and x2-x3 symmetry planes
  - \( \epsilon^{(1)} = \frac{C_{11} - C_{13}}{2C_{12}} \) and \( \epsilon^{(2)} = \frac{C_{11} - C_{33}}{2C_{33}} \);
- \( \delta^{(1)} \) — VTI parameter \( \delta \) in x1-x3 and x2-x3 symmetry planes
  - \( \delta^{(1)} = \frac{(C_{11} + 2C_{13} - C_{33} - C_{22})^2}{4C_{13}C_{33}} \) and \( \delta^{(2)} = \frac{(C_{11} + 2C_{13} - C_{33} - C_{22})^2}{4C_{33}(C_{33} - C_{22})} \);
- \( \gamma^{(1)} \) — VTI parameter \( \gamma \) in the x1-x3 and x2-x3 symmetry planes
  - \( \gamma^{(1)} = \frac{C_{44} - C_{55}}{2C_{44}} \) and \( \gamma^{(2)} = \frac{C_{55} - C_{66}}{2C_{44}} \).

Outside the symmetry planes, the generalized Thomasen parameters are computed by:

- \( \delta (\phi) \) — VTI parameter \( \delta \) outside of x1-x2 and x1-x3 symmetry plane \( \delta (\phi) = \delta^{(1)} \sin^2 \phi + \cos^2 \phi \);
- \( \epsilon (\phi) \) — VTI parameter \( \epsilon \) outside the x1-x2 and x1-x3 symmetry plane \( \epsilon (\phi) = \epsilon^{(1)} \sin^2 \phi + \epsilon^{(2)} \cos^2 \phi + (2\epsilon^{(2)} + \delta^{(3)} \sin^2 \phi \cos^2 \phi) \).

Velocity change study using synthetic data

The above method is used to model the velocity change under hydrostatic stress, uni-axial deformation and tri-axial stress tests with parameters taken from the literature (Prioul and Lebrat, 2004). Although the three principal strains of anisotropic medium are not necessarily equal under hydrostatic compression, an assumption is made that the principal strains are proportional to the (hydrostatic) stress for simplification. In the hydrostatic compression experiment, the principal strains in all three directions are -0.2%, i.e. the sample is shortened by 0.2% in x-, y-, and z-directions. In the uni-axial experiment, the sample decreases in height by 0.2%, whereas the lateral extent is not changed. In the tri-axial experiment, the sample is shortened by 0.2% under vertical compression and laterally extends 0.06% to simulate a Poisson ratio of 0.3. Since horizontal stress changes are the same, no azimuthally varying velocities need to be considered and we model and display results for velocity changes in Figures 3, 4 and 5 in the x2-x3 plane. Sensitivity parameters and initial stiffnesses are as follows (unit for stiffness tensor and TOE coefficients is GPa):

- \( C_{11} = -7034 \); \( C_{12} = 2147 \); \( C_{13} = 296 \);
- \( C_{11} = 33.9 \); \( C_{13} = 18.2 \); \( C_{01} = 32.1 \); \( C_{04} = 8.5 \); \( C_{66} = 14.2 \);
- \( \rho = 2700 \text{ kg/m}^3 \).

Figure 3 shows velocity changes under hydrostatic stress increase. The rock decreases in volume. Both P- and S-velocities in all directions increase as hydrostatic compressive stress increases. Slight changes in velocity anisotropy are also predicted, despite the applied isotropic strain. This is not surprising, as the sample is initially in an anisotropic state, but it serves as a reminder that anisotropic velocity changes can occur even under hydrostatic stress changes. The applied strain of 0.2% (-0.002) causes an increase in P-wave velocity slightly less than 20% (0.2).
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Figure 4: Relative velocity change under uni-axial strain.

Figure 4 shows velocity changes under uni-axial strain changes. Vertical stress compresses the rock and causes negative (contraction) strain in vertical direction. Strain in horizontal directions remains unchanged. Note that for horizontal propagation in all wave modes, there is an increase in velocity. For wave modes with horizontal polarization, velocity increases despite zero horizontal strain. This is caused by the contribution of vertical strain to $C_{11}, C_{22}, C_{44}$ and $C_{55}$. The 5% to 10% increase in velocities is less than the hydrostatic test, where velocity changes approaching 20% were predicted for the same amount of vertical strain.

Figure 5: Relative velocity change under tri-axial stress.

Figure 5 shows the predicted velocity change under tri-axial stress changes. The rock is simultaneously shortened in vertical direction, while expanded in horizontal directions. We chose the stress path (ratio between horizontal and vertical stress change) to closely simulate the stress path experienced during overburden stretching. However, in order to stay consistent with the two other experiments, we kept a vertical strain of -0.2% in compression, instead of extension and therefore applied 0.06% horizontal strain in extension, instead of compression as would be expected during overburden stretching. Again, the increase in vertical strain during the tri-axial experiment causes an increase of $V_p$ in vertical direction. The ratio of fractional (vertical) velocity change $\Delta V/V$ and vertical strain $\varepsilon_{33}$ (R-factor of Hatchell and Bourne and dilation parameter $\alpha$ of Røste et al., 2005) for the three experiments are R=60, 40 and 30 for the hydrostatic, uni-axial strain and tri-axial experiment, respectively. These values are in broad agreement with the laboratory observations reported by Bathija et al., 2009 and Holt et al., 2008. Both reports give experimental evidence of the influence of stress-path on vertical velocity and show that R-factor values in the range 1-100 are reasonable.

To summarize, the velocity change in any direction is the combined effect of compaction by all stresses applied. For example, the vertical P-wave velocity is influenced by both vertical and horizontal stress changes. The relative influence of vertical and horizontal strain on vertical velocity is given by (i) the magnitudes of vertical and horizontal strain and (ii) by the size of the stress-sensitivity coefficients $C_{111}$ and $C_{112}$. The three tests confirm that the propagation velocity typically increases if either propagation direction or polarization direction of the seismic wave is aligned with the direction of the applied compressive stress. The uni-axial strain experiment closely simulates the reservoir compaction. The velocity change computation depends on the TOE parameters from measurement. The predicted velocity changes are large enough to be using detectable by high quality seismic data. Therefore, observation of spatial and temporal seismic velocity changes is a possible monitoring tool for stress change. The shear wave, especially the pseudo shear wave is sensitive to horizontal stress, and is a good indicator for horizontal stress change.

Conclusions

The presented method provides a way to predict seismic velocity change where stress and strain changes are known. Therefore the work presented here is useful, in combination with reservoir geomechanical modeling, in forward modeling and feasibility studies. The rock-physics model can be used to predict subsalt velocity reduction as well as spatial variations in the anisotropic parameters (Sengupta et al., 2009) and time-lapse stress effects in seismic data (Herwanger and Horne, 2009). The work confirms the strong influence of both vertical and horizontal stress on seismic velocity and should act as a warning to only consider hydrostatic stress changes in seismic applications for stress-field monitoring.
EDITED REFERENCES
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REFERENCES


Røste, T., A. Stovas, and M. Landro, 2005, Estimation of layer thickness and velocity changes using 4D prestack seismic data: 67th Meeting, EAGE, Expanded Abstracts, C010


Sengupta, M., R. Bachrach, and A. Bakulin, 2009, Relationship between velocity and anisotropy perturbations and anomalous stress field around salt bodies: The Leading Edge, 28, no. 5, 260–266.


Thurston, R., 1974, Waves in solids in W. Truesdell, ed., Encyclopedia of physics: Springer-Verlag, VIa4