Crossline wavefield reconstruction from multi-component streamer data: multichannel interpolation by matching pursuit
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Summary
We introduce a new technique that uses multicomponent seismic measurements that would be recorded by a true multicomponent streamer to reconstruct the seismic wavefield at any desired position between streamers. This method, called Multichannel Interpolation by Matching Pursuit (MIMAP), operates on pressure and crossline particle motion measurements. As a data-dependent technique, MIMAP can interpolate severely aliased data without any assumptions about seismic events such as linearity or the model related to the seismic wavefield. MIMAP has the capability to perform well in the presence of irregular sampling and is robust even when only a small number of samples are available.

Introduction
In towed-streamer marine seismic acquisition, the crossline sampling can be irregular, and is typically coarse. In practice, only a limited number of streamers can be towed, and in order to acquire data over a wider aperture, or spread-width, the streamer separation is often greater than may be desired. Consequently, the recorded wavefield can be strongly aliased in the crossline direction and this can make reconstruction of the seismic wavefield on a regular 3D grid particularly difficult in practice.

One common proposal to interpolate data in the presence of aliasing is to rely on certain assumptions, such as linearity. Under this assumption, seismic events in time-space analysis windows are assumed to be linear, and consequently low frequency information is extrapolated to high frequencies where aliasing is present. This is the approach used in most of the interpolation/regularization techniques proposed in literature in the last two decades (Spitz, 1991; Gülünay, 2003; Liu and Sacchi, 2004; Zwartjes and Sacchi, 2007; Trad, 2009; Schonewille et al., 2009).

More recently, Robertsson et al. (2008) introduced the concept of a true multicomponent streamer, measuring the full particle velocity vector in addition to the pressure, and showed that the value of the horizontal component of the particle velocity measurement helps to address the issue of crossline sampling, increasing the crossline Nyquist wavenumber by a factor two (Linden, 1959). In fact, the equation of motion dictates that the particle acceleration vector (\( \mathbf{A} \)) is proportional to the gradient of pressure \( \mathbf{P} \):
\[
\nabla \mathbf{P} = -\rho \mathbf{A}.
\]
where \( \rho \) is the density of the medium. In particular, in crossline (\( y \)) direction, we have:
\[
\frac{\partial \mathbf{P}}{\partial y} = -\rho \mathbf{V} y.
\]
In this presentation, we examine crossline reconstruction of the seismic wavefield by processing the pressure and the crossline component of its gradient. To solve this problem we propose an iterative reconstruction technique called MIMAP (Multichannel Interpolation by Matching Pursuit). This reconstruction procedure is ideally suited for a limited number of samples that may be regularly or irregularly spaced.

In addition, as will be explained in detail, because of the different aliasing properties of the two measurements, the method works even when the data are aliased to a high-order, and it does not rely on assumptions such as the local linearity of the events.

MIMAP method
Multichannel Interpolation by Matching Pursuit (MIMAP) is a novel iterative waveform reconstruction technique for multicomponent data.

The method of matching pursuit was first proposed by Mallat and Zhang (1993) in the context of data compression. To achieve good compression, they sought to express the signal as a linear combination of a small number of parametric basis functions.

In the MIMAP method, we exploit a similar idea, but for multichannel wavefield reconstruction. Suppose that the samples of the seismic signal and its gradient are acquired at the spatial locations \( y_k, k=1,2,...,L \), which may be irregularly spaced. We model the underlying continuous seismic data \( f(y) \) as a sum of basis functions \( g(y;\theta_j) \) with parameter set \( \theta_j \):
\[
f(y) = \sum_j g(y;\theta_j),
\]
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The spatial gradients of the functions $f(y)$ and $g(y;\theta)$ are given by

$$f_j(y) = \frac{\partial}{\partial y} f(y), \quad g_j(y;\theta) = \frac{\partial}{\partial y} g(y;\theta).$$

In (3), the basis functions are chosen as continuous functions of space, so once they are determined, they can be evaluated at any desired position to reconstruct the signal. Basis functions that can be used in parameterization of seismic signals include sinusoids, complex exponentials, wavelets, and curvelets, amongst others.

At each iteration of the MIMAP algorithm, a new basis function is added to the model and then the modelling error waveform, i.e., the residual, is updated. To find the parameters of the basis function in MIMAP in the $P$-th iteration, a weighted combination of the residual energies corresponding to the data and their gradient is minimized:

$$\theta_P = \arg\min_{\theta_P} \sum_{k=1}^{P-1} |r_k(y_j)|^2 + \lambda \sum_{k=1}^{P-1} |r_k(y_j)|^2.$$  \hspace{1cm} (6)

For instance, when the basis functions are sinusoids, the parameter describing them are: the amplitude, $A_k$, the wavenumber, $k$, and the phase, $\phi_k$. The cost function to be minimized is the following:

$$\langle A, k, \phi \rangle = \arg\min_{\langle A, k, \phi \rangle} \left\{ \sum_k |r_k(y_j) - \lambda \cdot 2\pi k \cdot y_j + \phi_j|^2 + \lambda \sum_k |r_k(y_j) - \lambda \cdot 2\pi k \cdot y_j + \phi_j|^2 \right\}$$  \hspace{1cm} (7)

where $r_k(y_j)$ is the residual error on the derivative signal from the previous $P$-1 iterations.

As is clear by looking at equations (6) and (7), the parameter $\lambda$ balances the relative importance of the residuals of derivative and pressure in choosing the optimal parameters. The proper selection of the parameter $\lambda$ should take into account the energy difference between the two signals, but also their SNRs. In fact, it is chosen to force the algorithm to rely more on the quieter measurement than the noisier one.

When $\lambda$ is set to 0, MIMAP degenerates to IMAP, (Interpolation by Matching Pursuit: Özdemir et al., 2008, Özbek et al., 2009) and only pressure measurements are taken into account.

MIMAP de-aliasing capability

In this section, we discuss MIMAP’s capability to interpolate in the presence of high order aliasing. To understand this phenomenon, let us consider a monochromatic wave, i.e., a sinusoid, sampled on a uniformly spaced grid. If another sinusoid has the same amplitude and phase as the original, and its wavenumber differs from the first one by a multiple of the sampling rate, then these two sinusoids will have exactly the same samples on the uniform grid. This is known as aliasing.

When there is aliasing in single component acquisition, it is not possible to identify the correct waveform from the acquired samples. On the other hand, if the two sinusoids have the same samples on a uniform grid, then the gradients of these sinusoids cannot be the same on the regular grid. This is easy to see by observing that for a sinusoidal signal, the amplitude of the gradient is proportional to the amplitude and wavenumber of the signal. Hence, the samples of many sinusoids may fit to the data but among all those sinusoids, only the correct one will have a gradient that matches the gradient samples, as shown in Figure 1. Although this may not be always true for more complex wavefields, the sinusoid example illustrates in a simple and intuitive way the potential of MIMAP to interpolate beyond twice the Nyquist rate.

Figure 1: Diagram showing that two aliased sinusoids, identical at the sampling positions, have gradients with different amplitudes at the same position. This suggest that, despite the order of alias, a data dependent technique matching both signal and gradient can identify the correct wave amongst the possible aliased replicas.

When operating in the $f-y$ domain, and interpolating a frequency slice in the crossline direction, the basis function that best matches the data, at every iteration, is the basis function that minimizes the joint residual on the pressure and the particle velocity inputs. When aliasing is present, thanks to the property mentioned above, the only spectral
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replica that simultaneously matches the two input measurements is the correct one. The feature of MIMAP that we describe here is particularly important because it enables the reconstruction of high-order aliased data without relying on certain assumptions on the data, such as linearity. Under this latter assumption, seismic events in time-space analysis windows are assumed to be linear, and consequently, low frequency information is extrapolated to high frequencies where aliasing is present. This is a common approach used to solve the problem of spatial aliasing in most of the interpolation/regularization techniques proposed in literature in the last decades (Spitz, 1991; Gülünay, 2003; Liu and Sacchi, 2004; Zwartjes and Sacchi, 2007; Trad, 2009; Schonewille et al., 2009).

Example

To test MIMAP, we used a realistic synthetic dataset generated by finite difference modeling. The dataset simulates what would be recorded by a 3D multicomponent streamer survey over a geological environment representative of some areas of the Barents Sea, in the presence of shallow salt domes and a very complex structure. The source signature spectrum is flat; and its bandwidth is limited to 30Hz. Even with this limitation, the complexity of the model leads to the generation of a challenging dataset. The acquisition geometry consists of a regularly spaced receiver carpet on a 25m x 25m grid. Every receiver point records pressure and velocities in Cartesian coordinates. The signals, $P$ and $V_p$, have been decimated in the crossline direction at realistic crossline spacings (e.g. 125m). The MIMAP technique was applied on these data to reconstruct the data at 25m crossline sampling.

Figure 2 shows the high order de-aliasing performance of MIMAP: the pictures concentrate on a time slice of synthetics whose source signature bandwidth is up to 45Hz. Pressure and gradient data are decimated to 75m and severe aliasing is present. We interpolated the data to 12.5m spacing by using both, single and multichannel interpolators. It is evident that a single component standard interpolation (magenta line) cannot properly reconstruct the data. In fact, the main lobes of the dominating seismic event are completely missed by the sinc interpolator. Interestingly, the multicomponent sinc (Linden, 1959), cyan line, that is band-limited in a bandwidth up to the sampling rate, also fails to properly reconstruct the main event. In this case, the interpolation error energy looks even higher than in the single component case, despite the fact that at least the main event can now be detected. The anti-alias action of MIMAP (blue line) is clear in this example.

Figure 3 shows the crossline gather detail from a super-swath reproducing a realistic WAZ configuration (Moldoveanu et al., 2009), consisting of 36 streamers at 125m separation. Panel (a) shows the coarsely sampled pressure wavefield, used as input together with co-located $V_p$ traces, while the gather in panel (b) shows the reconstructed pressure wavefield obtained using MIMAP. Another view of the same data is given in Figures 4 and 5, showing a close-up of an area of interest and high complexity from the same dataset, shown in both the time-crossline and the frequency-wavenumber domains.
Conclusions

We introduced a new technique called MIMAP that uses multicomponent seismic measurements to reconstruct the seismic wavefield at any desired position between the streamers. MIMAP operates, simultaneously, on pressure and crossline particle motion measurements spaced at arbitrary locations. As a data-dependent technique, MIMAP takes special advantage of the different aliasing properties of the two measurements and interpolates high-order aliased data without making assumptions about seismic events, such as linearity. MIMAP is robust even when only a small number of samples are available. Using synthetic data of increasingly realistic geometry and complexity, we showed that the new method reconstructs signals sampled at crossline spacings for which the pressure-only data is irrecoverably aliased.

Acknowledgements

The synthetic dataset was generated as part of collaborative projects between Schlumberger, Lawrence Livermore National Laboratory, and StatoilHydro: we thank Shawn Larson of Lawrence Livermore National Laboratory and Martin Musil and Clement Kostov of Schlumberger. We also thank our colleagues Johan Robertsson, Dirk-Jan van Manen, Tony Curtis and Ralf Ferber for stimulating discussions, StatoilHydro for permission to show the synthetic data and Schlumberger for permission to publish this work.
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